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ABSTRACT

In San Diego and Imperial Counties (California) all high schools now require between 2 and 3 years of mathematics of their students. This represents almost a doubling of the minimum mathematics requirements in the last 2 years. While the requirements have been increasing, the availability of qualified secondary mathematics teachers has been decreasing. Over 70% of the mathematics teacher appointments made during the past 6 years were teachers already in the systems who transferred from other disciplines. The central goal of the University of California San Diego/San Diego State University (UCSD/SDSU) California Mathematics Project was to provide information useful in planning re-training programs for secondary mathematics teachers in San Diego and Imperial Counties. This report specifies the current status of secondary mathematics teaching in this area. Discussions include a profile of mathematics teachers, a follow-up of the re-training program, and program evaluation. Included are 18 tables and 18 references. Appendices include the district survey, teacher survey, and information about the Mathematics Diagnostic Testing Project exams. (DC)

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Report #5

SECONDARY MATHEMATICS EDUCATION IN SAN DIEGO AND IMPERIAL COUNTIES

California Mathematics Project
Final Report
31 July 1984

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Laura Martin
Steve Black

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La Jolla, California

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Report #5

SECONDARY MATHEMATICS EDUCATION IN SAN DIEGO AND IMPERIAL COUNTIES

California Mathematics Project
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Randall Souviney
Laura Martin
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University of California, San Diego
La Jolla, California

This report summarizes the activities of the joint UCSD and SDSU California Mathematics Project (Grant #UCSD 5052), directed by Alfred Manaster and Randall Souviney at the University of California, San Diego, and Douglas McLeod at San Diego State University.

TABLE OF CONTENTS

Acknowledgements iv
List of Tables v
Executive Summary. vii

I. INTRODUCTION. 1

The UCSD/SDSU Study 4

II. PROFILE OF CURRENT MATHEMATICS TEACHERS 6

The UCSD District Survey 6

The UCSD Teacher Survey. 7

Retraining Considerations. 14
District Certificate Programs. 16
Identifying "Cross-over" Teachers. 19
Alternate Ways to Count "Cross-over" Teachers. . . . 19
Recent Mathematics Appointments. 22
Continuing Education for Qualified Teachers. 27

III. OBSERVATION COMPONENT 29

Overview 29

Problem Solving as a Focus in the Classroom. 32
Group Problem-Solving 34
Effects of the Summer Program 35
The Nature of Mathematics 41

Redefining Basic Skills. 42
Use of New Materials: Calculators and Computers . . . 43
Other New Materials 45
New Techniques. 47
The Use of Mathematical Language. 51

Applying Stringent Standards 53
Specific Motivation 55
Socio-Economic Factors. 56
Establishing and Shifting Innovations 58
Constraints on Efficiency 61
Mathematics as a Subject Matter. 61

Evaluation Using a Wide Range of Measures. 64
Local Evaluations 67
Observed Assessment Measures. 68
Use of the MDTP Instruments 70
Effects of the Program. 71

Expanding Range and Quantity of Mathematics Offerings. . 71
New Directions in Mathematics Curriculum. 73
Flexibility in Using Materials. 75

Encouraging Professionalism. 76
Participating in the CMP Institute. 76
Leadership Training 77
Other Influences on Professionalism 79
The Electronic Message System 82
The Two Options 84

Public Support for Mathematics Instruction 85

IV. MATHEMATICS DIAGNOSTIC TESTING PROJECT. 88

V. CONCLUSION. 90

References. 92

Appendix A: The UCSD District Survey. 95
Appendix B: The UCSD Teacher Survey 99
Appendix C: The Mathematics Diagnostic Testing Project. . . 102

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LIST OF TABLES

Table 1. Secondary Mathematics Teaching Loads
According to the UCSD Teacher and CBEDS Surveys (%)

Table 2. Age of Secondary Mathematics Teachers
According to the UCSD District and CBEDS Surveys (%)

Table 3. Highest Degree Earned by Secondary Mathematics Teachers
According to the UCSD Teacher and CBEDS Surveys(%)

Table 4a. Total Teaching Experience of All Secondary Mathematics Teachers
According to the UCSD Teacher Survey (%)

Table 4b. Total Teaching Experience of All Secondary Mathematics Teachers
According to the CBEDS Survey (%)

Table 5a. Credential Type for Secondary Mathematics Teachers
According to the UCSD Teacher and District Surveys (%)

Table 5b. Credential Type for Secondary Mathematics Teachers
According to the CBEDS Survey (%)

Table 6. Mathematics Teaching Experience by Credential Type
According to the UCSD Teacher Survey (%)

Table 7. Mathematics and Total Secondary Teaching Experience
According to the UCSD Teacher Survey (%)

Table 8. Highest Mathematics Degree Earned by Secondary Mathematics Teachers
According to the UCSD Teacher and District Surveys (%)

Table 9. Current Secondary Mathematics Teachers Not Meeting
Selected Criteria (%)

Table 10. Secondary Teachers Responsible for One or More Sections
of College Preparatory and General Mathematics Courses
According to the UCSD Teacher Survey (%)

Table 11. Secondary Mathematics Teachers of Each Appointment Type
According to the UCSD Teacher Survey (%)

Table 12. Recently Appointed Secondary Mathematics Teachers With
Less Than a Minor or Major in Mathematics
According to the UCSD Teacher Survey (%)

Table 13. Credential Type of Recent Secondary Mathematics Appointees With Less Than a Minor or Major in Mathematics According to the UCSD Teacher Survey (%)

Table 14. Recent Appointees and All Secondary Teachers Responsible for College Preparatory Courses According to the UCSD Teacher Survey (%)

Table 15. Mathematics Background for Junior and Senior High Mathematics Appointees Teaching Algebra and Higher Level Courses According to the UCSD Teacher Survey (%)

Table 16. Measures of Student Performance

UCSD/SDSU California Mathematics Project

EXECUTIVE SUMMARY

Several national reports published in recent years have made wide-ranging recommendations for improving the quality of our schools. Each recommends increasing the quality and quantity of mathematics required of all secondary school graduates. In San Diego and Imperial Counties, all high schools now require between two and three years of mathematics of their students. This represents almost a doubling of the minimum mathematics requirements over the last two years. The minimum mathematics requirement for university entrance has increased by one additional year as well.

While these requirements have been increasing, the availability of qualified secondary mathematics teachers has been decreasing. Our surveys indicated that only about 34% of the secondary mathematics teacher appointments made during the past six years have been filled by individuals possessing appropriate credentials and adequate mathematics background. The distribution of credentials and appointment types among new-to-mathematics teachers showed that over 70% of these appointments were teachers already in the system. They transferred from other disciplines and were most in need of upgrading in mathematics.

The central goal of the UCSD/SDSU California Mathematics Project was to provide information useful in planning re-training programs for secondary

mathematics teachers in San Diego and Imperial Counties. A District Personnel Survey and an individual Teacher Survey were conducted to obtain background information on mathematics teachers' preparedness and experience. The returns on each survey yielded data for approximately 56% of the area's mathematics teachers. Comparisons with the California Basic Education Data System (CBEDS) results showed that the UCSD Surveys were reasonably representative of the population of secondary mathematics teachers in the two counties.

In San Diego and Imperial Counties, approximately 60% of the mathematics teachers have full-time mathematics teaching assignments. Half of all secondary mathematics teachers have less than 10 years mathematics teaching experience. The most frequently reported level of mathematics teaching experience was 1-3 years. Finally, approximately 47% of all mathematics instructors teach at least one section of a college-preparatory course (basic algebra and higher level courses). These figures together indicate that a significant proportion of teachers are new-to-mathematics instruction, teach mathematics on a part-time basis and have transferred to their current position from non-mathematics departments within the same district. Many of these teachers are responsible for college-preparatory as well as general and career mathematics courses.

The academic mathematics backgrounds of teachers is considered a critical factor for the quality of secondary mathematics programs. The National Council of Teachers of Mathematics recommends that Junior High mathematics teachers possess a minimum of an academic Minor in mathematics and at the Senior High level, a minimum of a mathematics Major. The UCSD District Survey showed that 44% of all grade 7-12 mathematics teachers are listed by their districts as having less than a college Minor in mathematics,

while the results of the UCSD Teacher Survey showed only 20% reporting less than a Minor. This discrepancy may be accounted for in two ways: (1) District Personnel offices often do not maintain an accurate record of academic Minors and (2) some teachers may have reported "informal" Minors based on mathematics courses taken since graduation. Both Surveys indicated that about 36% of all secondary mathematics teachers have a mathematics Major and more than 5% have earned a Master's degree in mathematics.

We estimate that total of about one-third of the approximately 1265 teachers in San Diego and Imperial Counties may need to upgrade their mathematics knowledge. Since many of these teachers with limited formal mathematics backgrounds may have acquired expertise from teaching mathematics over the course of many years, their numbers could be subtracted from this total in arriving at an estimate of the number of teachers in need of immediate re-training. Of those mathematics teachers appointed in the last six years, we estimate that about 250 currently require mathematics content and instructional methods upgrading. Our results indicate that nearly 13% of these new-to-mathematics teachers who reported less than a Minor in matvhematics, are responsible for one or more sections of college-preparatory mathematics. Overall, about 5% of the teachers responsible for one or more sections of a college-preparatory course have less than a Minor in mathematics.

Among the plans available to teachers for improving and upgrading mathematics skills are the CMP Summer Institute, courses at local colleges, universities, and TEC Center, and three district-administered re-training programs organized in conjunction with community colleges. Two of the district programs lead to local certificates; only one is approved by the

California Commission on Teacher Credentialing to recommend for issuance of Supplementary Introductory Mathematics authorizations. Approximately 90 teachers will be participating in these programs in San Diego County durng 1984-85. It is estimated that a two-year program of study leading to an academic Minor and a Supplementary Authorization to an existing credential could reasonably be mounted by local Universities and School Districts to accommodate up to 100 teachers per year. Such a program could provide for the effective upgrading of these new-to-mathematics teachers in time to do some good.

Plans to offer continuing education for teachers already fully qualified to teach secondary mathematics include the CMP Summer Institute and the district level Mentor Teacher Program. An extended year appointment plan for qualified mathematics teachers should be considered as a way to more fully utilize existing talents in the areas of program development, inservice leadership, research, and teaching.

In addition to survey data, observations and interviews were conducted during the 1983 CMP Summer Institute with participants and throughout the year with a small number of participants. A summary of conclusions made from this corpus of data is listed below.

Problem-solving will be a curriculum focus in school mathematics during the 1980's. Problem-solving activities have a powerful appeal to teachers of all levels of experience and background, but those with less mathematics teaching experience and academic background find it somewhat harder to integrate the activities into the standard curriculum. If problem-solving is to become an effective component within the mathematics

curriculum, it will need to be incorporated into the standard instructional materials teachers use.

The use of small, mixed-ability groupings for problem-solving and other mathematics activities is being encouraged by mathematics educators. This innovation was less likely to be maintained as an instruction component by teachers than other new instruction strategies. Reportedly, this was due to fact that the social re-structuring tended to result in off-task behavior on the part of students. Large classes, which were the rule, also presented difficulties for the successful introduction of small-group activities.

Computers were not integrated into mathematics instruction. Administrators and teachers tend to regard computers as a separate content domain and, in general, believe that mathematics teachers must learn how to teach programming before they can effectively use the technology in their teaching. Recent research indicates this not to be the case. Another problem preventing effective instructional applications of computers was the lack of emphasis during the Institute on methods for structuring the use of one machine for 35 students. However, as a result of the Institute, the participants seemed to embrace the wide-scale use of calculators within the mathematics curriculum.

Other new materials and activities introduced to teachers during the summer tended to be used at the beginning of the year, then to drop off in use, and finally to be re-introduced on a less frequent basis in the Spring. More experienced mathematics teachers of all backgrounds were more flexible in the use of new materials and in improvising upon standard text materials.

The likelihood that new materials introduced in training programs would be integrated into classroom instruction seemed more a function of how much time was spent using the new materials during training, rather than the inherent interest in or innovative nature of the materials. Most summer participants felt that a methods course organized around the grade-level taught rather than around the participants' backgrounds would have been useful. This has been arranged for the 1984 summer program.

Responding to student needs was regarded by the participants as an important component in effective teaching. At the Junior High level, teachers must often rearrange large lesson segments to accommodate the wide range of student achievement. Since teachers with less advanced mathematics backgrounds are often assigned to these introductory courses, their training should include a focus on flexibility within lesson presentations. All teachers were seen to be flexible in adapting lessons in response to feedback from students. However, the effectiveness of these shifts in lesson organization was positively related to the number of years teaching mathematics.

The most prevalent classroom instructional technique is modelling of appropriate responses by the teacher, including inferential thinking. Emphasis in teacher education should be placed on methods for encouraging more active participation of students in both teacher-directed and small group lessons.

Student motivation, socio-economic level, teaching schedules, and class size all interact with teaching style and the way a teacher views their teaching. Class size and range of students' abilities particularly affect the

performance of new-to-mathematics teachers.

The tendency of all teachers to fall into routines, both desirable and undesirable, is strong. If change in practice is warranted and desired by the teacher, these practices may be mitigated by the systematic presence of professionally supportive observers in the classroom; by having the teachers reflect on their own practices on a regular basis; and by frequent professional contacts with colleagues.

Non-traditional measures of change in students and teachers, though reasonably easy to recognize, are somewhat harder to specify objectively. Teachers tend to judge their effectiveness by relying on feedback from colleagues and by a "sense" of having communicated well during a lesson. Until alternative criteria can be objectively specified, test results and teacher intuition will continue to be the most prevalent form of student evaluation.

Developing professionalism among teachers is vital to the health of school mathematics programs. The CMP Institute contributed greatly in this regard by providing collegial contact throughout the year and by regarding the participants' feedback as vital to its success. Leadership activities, although indirectly promoted during the 1983 session, were a direct outcome of the Institute. An extended leadership emphasis has been planned for the 1984 SDSU CMP Institute.

Job satisfaction does not seem to depend on having access to progressive instructional models, although a feeling of professionalism may. Job satisfaction depends on factors like control in the classroom, effectiveness

as an instructor, the feeling of being a resource for colleagues, and monetary reward. More emphasis during training should be placed on issues of classroom management. More resources should be allocated by schools and districts for participation in professional activities. Uncertainty about scheduling and class assignments is detrimental to teacher dedication, especially for those teachers undergoing a change in teaching assignment and subsequent upgrading of content knowledge.

The San Diego community is effectively working towards a support network for improved mathematics instruction through inter-segmental cooperation. Training and dialogue for and among experienced and non-experienced secondary teachers is a significant activity promoted by the CMP Institute. Parent involvement is also being encouraged at the school level, in part, through the use of the Mathematics Diagnostic Testing instruments.

Recommendations

1. The California Mathematics Project should consider encouraging local Projects to become directly involved in the re-training of new-to-mathematics teachers who have limited formal training in mathematics. The San Diego State University CMP Institute can serve as a model in this regard.

2. Inter-segmental cooperation within the education community can significantly increase the efficiency of retraining programs. Universities, County Departments of Education, TEC Centers and local Districts should cooperate in the development of re-training programs which lead to at least an academic Minor in mathematics and CTC Supplementary Authorization to existing credentials.

3. The California Mathematics Project should encourage local Projects to identify highly qualified secondary mathematics teachers for leadership training. These teachers should participate actively in the upgrading of content and methods knowledge for new-to-mathematics colleagues. CMP Internships and cooperation with existing Mentor Teacher Programs should be explored as possible alternatives.

4. The California Basic Education Data System (CBEDS) teacher questionnaire should be expanded to include questions which will provide more detailed information on teachers' academic backgrounds and the type of credential authorization used for their current teaching appointment.

I. INTRODUCTION

Once again our society is calling for an increase in the quality of education, especially in the areas of mathematics, science and technology. The rationale behind this current concern with the quality of education is industrial production, information technology, and national security. We have seen similar appeals before. After each major war of this century and in response to the Soviet launch of Sputnik in 1957, major reviews of schools and schooling have been called for by government, industry and the military.

Our society, now increasingly dependent on technology, must examine closely the type and quality of education in our schools in order that it serve our youth well into the next century. Applications of technology will be likely to impact all employment sectors. The service industry will be especially influenced. It is projected to offer the greatest number of new job opportunities in the future.

Mathematics and its role in solving problems plays a central role in the development of control over information technology for <u>all</u> students. Recent reports have suggested that an apparent scarcity of qualified mathematics teachers in secondary classrooms is having a debilitating effect on efforts to increase the quantity and quality of mathematics courses offered in grades 7-12. In an attempt to obtain systematic information on this topic, this report will attempt to specify more accurately the current status of secondary mathematics teaching in the San Diego and Imperial County area of the State of California.

Several reports of national scope have been published during the past two years which make wide-ranging recommendations for improving the quality of our schools (NSF, 1982 & 1983; Adler, 1982; Boyer, 1983; Peterson, 1983; NCEE, 1983; TFEEG, 1983). Though differing in specifics, each recommends significant increases in the quality and quantity of mathematics required of all secondary school graduates. The California State Board of Education report (1983), Raising Expectations: Model Graduation Requirements, recommends changing the high school graduation requirement for mathematics from 2 years to 3 years. The suggested content of these courses is algebra, geometry, statistics and probability. Slightly more than half (55%) of the mathematics courses offered in San Diego and Imperial Counties are general math, designed for students who will move directly into the workplace upon graduation, The remaining 45% of the sections are for college preparatory students (first year algebra or above). The State-wide figures are 61% general math and 39% college preparatory (CBEDS, 1982-83).

Concurrent with a recommended increase in overall mathematics competence in the high schools, university entrance requirements are increasing. The University of California will require three years of high school mathematics for entering freshmen in 1986. The California State University system will require a minimum of two years of high school mathematics for entering freshmen in the Fall of 1984. There is pressure from various CSU campus constituencies to increase this entry requirement to three years. Together, these two policy decisions are likely to increase dramatically the number of sections of college preparatory mathematics needed to teach students in grades 10-12.

Paradoxically, at a time of increasing high school graduation and college entry requirements, the availability of qualified secondary mathematics teachers is decreasing. For example:

1. The UC and CSU systems had less than 100 students enrolled in mathematics teaching programs in 1981 (Guthrie and Zusman, 1982).

2. During 1984, San Diego area colleges and universities reported graduating a total of 16 newly qualified candidates for Single Subject Mathematics Credentials.

3. Over the past three years, the number of Preliminary (initial) Single Subject Mathematics Credentials has been gradually decreasing to a statewide total of 228 in 1983.

4. Though the number of Clear (full) Credentials has about doubled to 328 over the past three years, individuals applying for these credentials are generally practicing teachers "clearing" a Preliminary Credential or renewing an existing Clear Credential rather than new teachers entering the field.

5. The California Commission for Teacher Credentialing reports that in response to district needs, the number of long-term emergency Single Subject Mathematics Credentials has more than doubled over the past three years to 430 in 1983.

6. In 1981, only about 2450 newly qualified mathematics teachers throughout the country (about half the total trained) sought and received full time employment as teachers (Feistritzer, 1983).

The UCSD survey of San Diego and Imperial County secondary mathematics teachers indicated that only about 34% of the approximately 425 grade 7-12 mathematics teacher appointments made over the past six years have been filled with individuals possessing appropriate credentials and adequate mathematics backgrounds. The shortfall was made up in large part by teachers transferred from other disciplines within districts. An already discouraging situation with regard to the availability of qualified mathematics teachers has been exacerbated by a virtual freeze in secondary teacher hiring due to a drop in secondary school enrollments since 1977 and an erosion of school

budgets. Salary competition from industry has also been a negative factor in recruiting qualified teachers even when positions became available.

In response to this increasingly grim state of affairs, in 1983 the California State Legislature established the California Mathematics Project (CMP) as one mechanism for increasing the quality of K-12 mathematics instruction. Eight CMP Institutes were established throughout the State at various University of California and California State University sites. The CMP Institute at San Diego State University (SDSU), in cooperation with the Department of Education, San Diego County (DOESDC), was unique among the projects funded in that it provided inservice opportunities for both highly qualified and new-to-mathematics teachers. The current study, in part, assesses the potential of such retraining efforts as a means for redressing the current shortage of qualified mathematics teachers.

The UCSD/SDSU Study

The UCSD/SDSU California Mathematics Project conducted a three component study of secondary mathematics teaching in San Diego and Imperial Counties. Each element of the study contributed to our understanding of preservice and inservice training requirements for secondary mathematics teachers. The three project components are summarized below:

1. In cooperation with the San Diego State University (SDSU) and the San Diego County Department of Education (SDCDOE), we observed the participation of approximately 50 secondary mathematics teachers during the SDSU CMP Summer Institute. Approximately half of the participants had strong backgrounds in mathematics and half had less than a Minor in mathematics. Selected participants were observed in their classrooms throughout the 1983-84 school year. Case studies were prepared on the effects of Institute activities on subsequent teaching practices in the six classrooms.

2. Two surveys of teacher assignments, workloads and professional qualifications were conducted in San Diego and Imperial Counties. The first surveyed was based on Personnel records at the district level. The second was based on information from each grade 7-12 mathematics teacher in the two counties.

3. Through the use of Mathematics Diagnostic Testing Project instruments, the Project documented achievement profiles of students in schools serviced by Institute participants and other interested mathematics teachers.

In the following section we will review the results of the UCSD District Personnel and Teacher Surveys. These surveys provide an overall profile of the professional backgrounds of current secondary mathematics teachers in San Diego and Imperial Counties. Section III presents a summary of the observational component of the study and Section IV reviews the results of the Mathematics Diagnostic Testing Project analysis.

II. PROFILE OF CURRENT MATHEMATICS TEACHERS

The UCSD District Survey

A survey of teacher assignments, workloads, and professional qualifications was conducted to document current levels of teacher qualifications and projected personnel requirements for grades 7-12 (see Appendix A). In May of 1983, the UCSD District Survey was mailed to all Personnel Directors in districts serving students in grades 7-12 in San Diego and Imperial Counties. Due to a poor response rate over the summer, it was mailed again in September 1983. Overall, 24 of the 43 districts responded which represents about 60% of the mathematics teachers in the two county area. Two of the three largest school districts in the county, San Diego Unified School District and Grossmont High School District, are represented. The second largest district reported that it was unable to participate due to the administrative workload.

This survey showed that the minimum mathematics graduation requirement for most districts was two years. Several districts have increased this requirement to a minimum of 3 years of mathematics. Personnel directors report that nearly 60% of the mathematics teachers in grades 7-12 have at least an academic Minor in mathematics. The median years of teaching experience ranged from 10 to 13 years and the median age ranged from 41 to 45. About 32% of the teachers held a General Secondary Credential (issued through the late 1960's), 39% held a Standard Secondary Credential (issued through the late 1970's), 18% held the current Single Subject Credential, and

the remainder held a Multiple Subjects (current elementary), other Elementary, Emergency or Other credential.

The UCSD Teacher Survey

In January 1984, all the public Junior and Senior High Schools in San Diego and Imperial Counties were mailed surveys to be completed by teachers responsible for at least one mathematics section per day (see appendix B). 720 forms were returned. The California Basic Education Data System Survey (CBEDS) 1982-83 data indicate that 1265 local teachers reported teaching at least one section of mathematics per day in grades 7-12. There were 1180 mathematics teachers in San Diego and 85 in Imperial County. Using this 1982-83 figure as an estimate for the total number of mathematics teachers who received surveys, the CMP Teacher Survey achieved about a 57% rate of return.

Based on the 1983-84 CBEDS data, San Diego and Imperial Counties accounted for about 9% of the total secondary mathematics teaching effort in the State of California. Table 1 shows that approximately just over half (54%) of the secondary mathematics teachers taught mathematics full-time (5 or more sections per day). Over 25% taught 1 or 2 sections per day. The pattern was similar when considering Junior (7-9) and Senior (10-12) High teachers separately. The slightly higher percent of full-time mathematics teachers reported in the UCSD Teacher Survey over the 1983-84 CBEDS data and a lower percent of teachers responsible for only one mathematics section, may skew our results in favor of the more qualified mathematics instructors. As a result, the quality measures reported in the following tables may be somewhat higher than is actually the case.

Table 1

Secondary Mathematics Teaching Loads
According to the UCSD Teacher and CBEDS Surveys(%)

	Percent of Time Teaching Mathematics				
	0-20%	21-40%	41-60%	61-80%	81-100%
UCSD Teacher Survey					
Total (n=712)	10.1	11.0	9.0	9.0	61.0
CBEDS (1983-84)					
San Diego/Imperial (n=972)	14.4	11.3	11.3	9.0	54.0
Total State (n=10976)	17.9	12.6	10.1	9.4	50.1

NOTE: 1983-84 CBEDS data reported for this table only.

As shown in Tables 2, 3, 4a and 4b, the medians and distributions of the demographic variables -- age, highest degree and teaching experience -- were similar for the UCSD District Survey, the UCSD Teacher Survey and the CBEDS Survey. These results added to our confidence that our results can be interpreted as a reasonable representative of the population of San Diego and Imperial County secondary mathematics teachers.

Table 2

Age of Secondary Mathematics Teachers
According to the UCSD District and CBEDS Surveys (%)

	Age in Years								
	20-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	60+
UCSD District Survey									
SD/Imperial (n=722)	1.0	7.2	14.5	20.2	15.8	12.3	12.9	9.9	5.8
CBEDS (1982-83)									
SD/Imperial (n=1265)	2.4	9.5	17.5	19.3	13.4	11.9	12.3	9.7	4.1
Total State (n=16441)	1.8	8.4	16.9	17.8	14.9	13.2	13.0	9.5	4.5

Table 3

Highest Degree Earned by Secondary Mathematics Teachers
According to the UCSD Teacher and CBEDS Surveys (%)

	Degree		
	BA	MA	PhD
UCSD Teacher Survey			
SD/Imperial (n=701)	41.9	56.3	1.7
CBEDS (1982-83)			
SD/Imperial (n=1265)	45.1	52.5	2.2
Total State (n=16441)	53.4	44.2	1.0

Table 4a

Total Teaching Experience of All Secondary Mathematics Teachers According to the UCSD Teacher Survey(%)

	Years Experience									
	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24	25-27	28+
UCSD Teacher Survey										
(n=712)	7.0	8.6	13.2	11.9	15.9	10.5	6.7	9.7	6.2	10.3

Table 4b

Total Teaching Experience of All Secondary Mathematics Teachers According to the CBEDS Survey(%)

	Years Experience								
	0-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	40+
CBEDS (1982-83)									
SD/Imperial (n=1265)	16.9	19.1	25.7	14.5	13.2	7.0	3.2	0.2	0.1
Total State (n=16441)	12.2	18.2	21.6	18.0	15.5	8.9	4.9	0.5	0.2

Tables 5a and 5b show that slightly more than 30% of both the Junior and Senior High mathematics teachers possessed General Secondary credentials (issued until the late 1960's and authorizes holders to teach all subject areas regardless of academic background). Approximately 44% of the Junior High teachers and 58% of the Senior High teachers reported having a Standard Secondary Mathematics Credential, a Ryan Single Subject Mathematics Credential, a Supplementary Introductory Mathematics Authorization or a Supplementary Multiple Subjects Authorization (each of these authorize holder to teach mathematics based on extent of mathematics background). The remaining 24% of the Junior High and 10% of the Senior High teachers reported Annual Board Resolutions (School Board authorize holder of non-mathematics credential to teach mathematics on temporary basis), Emergency, Elementary or

Other authorizations such as Administration, to teach mathematics.

Table 5a

Credential Type for Secondary Mathematics Teachers According to the UCSD Teacher and District Surveys (%)

	General Second'y	Standard Second'y	Board Resol.	Ryan-SSub.	Ryan-Supp.	Multiple Subject	Elementary	Emergency	Other
UCSD Teacher Survey									
Jr Hi (n=322)	30.7	26.6	1.9	11.2	5.9	3.1	13.7	2.5	4.7
Sr Hi (n=385)	34.0	42.6	1.6	12.7	2.3	0.3	N/A	2.9	3.6
Total (n=707)	32.5	35.2	1.7	12.0	4.0	1.6	6.2	2.7	4.1
UCSD District Survey									
Jr Hi (n=304)	25.7	31.3	0.0	21.4	0.0	2.3	14.5	1.6	3.3
Sr Hi (n=440)	36.1	43.6	0.0	14.8	0.0	0.0	0.7	1.6	3.2
Total (n=744)	31.9	38.6	0.0	17.5	0.0	0.9	6.3	1.6	3.2

Table 5b

Credential Type for Secondary Mathematics Teachers According to the CBEDS Survey (%)

	Elementary	Secondary	Designated Subject	Other
CBEDS (1982-83)				
SD/Imperial (n=1273)	15.0	79.6	3.3	2.1
Total State (n=16441)	23.9	69.1	4.0	3.1

Table 6 shows that teachers with 6 or fewer years of mathematics teaching experience tend to possess a greater proportion of emergency and other non-mathematics teaching credentials than teachers with greater

mathematics experience. Teachers with 7-21 years of mathematics experience tend to have Standard and Ryan Single Subject credentials. The most experienced mathematics teachers possessed mostly General Secondary Credentials which were issued during the 1960's. It is likely that these teachers have been teaching mathematics for their entire careers.

Table 6

Mathematics Teaching Experience By Credential Type According to the UCSD Teacher Survey (%)

	Years Teaching Mathematics									
	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24	25-27	28+
Gen. Sec.	14.0	11.4	15.0	19.5	15.1	53.2	64.7	71.4	80.8	86.1
Stand. Sec.	9.0	22.6	51.4	62.2	72.0	34.0	23.5	14.3	15.4	8.3
Board Res.	2.2	6.6	0.9	0.0	0.0	0.0	0.0	2.9	0.0	0.0
Ryan-SSub.	12.7	22.6	22.4	4.9	8.6	6.4	8.8	8.6	0.0	0.0
Ryan-Suppl.	12.7	8.5	0.0	2.4	0.0	0.0	0.0	0.0	0.0	0.0
Mult.Subj.	6.0	0.9	0.0	2.4	0.0	0.0	0.0	0.0	0.0	0.0
Elementary	9.7	8.5	3.7	8.5	3.2	6.4	2.9	2.9	3.8	5.6
Emergency	11.2	1.9	1.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Other	12.7	3.8	4.7	0.0	1.1	0.0	0.0	0.0	0.0	0.0

(n=700)

As shown in Table 7, the median total teaching experience for secondary mathematics teachers was 13-15 years. The median drops to 10-12 years for mathematics teaching experience. The most frequently reported years of total teaching experience was also 13-15 years, however the the range drops to 1-3 years for mathematics teaching experience. Nearly 19% of the teachers began teaching mathematics in the past 3 years. Almost 35% have been teaching mathematics for 6 years or less. We will take a closer look at these new-to-mathematics teachers in a later section.

Table 7

Mathematics and Total Secondary Teaching Experience
According to the UCSD Teacher Survey (%)

	Years Experience									
	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24	25-27	28+
UCSD Teacher Survey										
Mathematics (n=703)	19.5	15.2	15.2	11.7	13.2	6.7	4.8	5.0	3.6	5.1
Total (n=712)	7.0	8.6	13.2	11.9	15.9	10.5	6.7	9.7	6.2	10.3

The UCSD Teacher Survey data shown in Table 8 indicate that over 80% of the mathematics teachers responding had earned a minimum of a Minor in mathematics (37% Mathematics Minor; 36% Mathematics Major; 8% MA in Mathematics).

Table 8

Highest Mathematics Degree Earned by Secondary Mathematics Teachers
According to the UCSD Teacher and District Surveys (%)

	Highest Math Degree			
	Minor	Major	MA	None
UCSD Teacher Survey				
Jr Hi (n=321)	40.2	32.1	4.0	23.7
Sr Jr (n=387)	33.9	38.8	10.6	16.8
Total (n=708)	36.7	35.7	7.6	19.9
UCSD District Survey				
Jr Hi (n=299)	17.1	31.1	1.0	50.8
Sr Hi (n=443)	13.3	40.0	7.2	39.5
Total (n=750)	14.7	36.0	4.7	43.6

The results for mathematics background are somewhat higher than those indicated by the UCSD District Survey or in the preliminary results reported earlier by Silver (1983). At the junior high level, 40% of the teachers

reported having a mathematics Minor, 32% reported a mathematics Major and 4% indicated they had an MA in mathematics. At the Senior High level, about 34% reported mathematics Minors, 39% Majors and about 11% MA degrees in mathematics. There were no PhD's in mathematics reported.

The difference in minimum mathematics background between the UCSD Teacher and District Surveys may be accounted for by the fact that personnel officers have incomplete records regarding the academic backgrounds of the teachers in their employ since degree information is verified only for the purposes of credentialing. Since many teachers choose to teach subjects in their academic Major area, academic Minors may go unreported. Also, some teachers participating in the UCSD Teacher Survey may have reported "informal" Minors based on the number of mathematics courses taken as undergraduates in universities not requiring a formal Minor course-of-study. As indicated earlier, there may also be some bias towards more experienced teachers in the UCSD Teacher Survey data due to self-selection.

Retraining Considerations

The California Commission for Teacher Credentialing (CTC) reports four alternate means for teachers with existing qualifications to receive authorization to teach mathematics in Grades 7-12:

1. Holders of the Standard Secondary or Ryan Single Subject Credential in a non-mathematics area or the Ryan Multiple Subject Credential can receive full mathematics authorization by passing the National Teacher Exam in mathematics;

2. Holders of the Ryan Single Subject Credential in a non-mathematics area, can qualify for a Supplementary Authorization--Introductory Mathematics (mathematics commonly taught in Grades 7-9) by completing the following requirements: 20 semester units (or 10 upper division semester units) of course work which includes a basic introduction to mathematics, college algebra, geometry and first year calculus.

3. Holders of the Ryan Multiple Subjects Credential can qualify for a Supplementary Mathematics Authorization (for Grades 7-9) by completing the following requirements: 20 semester units (or 10 upper division semester units) of course work which includes the development of the real number system, algebra, and geometry.

4. Individuals possessing a baccalaureate degree with an academic Major or Minor in mathematics, passing marks on the NTE mathematics subject area and CBEST exams, and a Certificate of Clearance can apply for a Teacher Trainee credential qualifying them to teach for two years in a district with a declared shortage of secondary mathematics teachers.

It is important to note that holders of the General Secondary Credential (32% of all grade 7-12 mathematics teachers reporting) are not legally obligated to pursue additional authorizations even if they have no college mathematics background. Approximately one-fourth of the teachers with General Secondary Credentials who teach mathematics in the San Diego and Imperial Counties reported having less than a minor in mathematics (nearly 8% of the total mathematics teaching force or about 100 teachers in San Diego County). In order to address the needs of these and other underqualified teachers of mathematics, several larger districts have established "certificate" programs

in cooperation with local community colleges.

Two of the local certificate programs are restricted to in-district certification since neither leads to additional Commission for Teacher Credentialing (CTC) authorization. They demonstrate Districts' commitment to improving the quality of mathematics instruction in their schools. Since many of the teachers served by these programs already have a General Secondary Credential and are not legally bound to seek additional authorizations, districts are going beyond minimum CTC requirements in the name of improved quality of instruction.

All the locally formulated certificate programs however, fall short of CTC Supplementary Authorization requirements: no calculus is required and several of the courses offered would not qualify for transfer to a four year institution. Discussions with district personnel officers revealed that teachers who need additional content instruction question the utility of requiring a course in calculus for the Supplementary Authorization to teach Introductory Mathematics in grades 7-9. These teachers are willing, therefore, to limit their career mobility by participating in a less demanding District sponsored certificate program. As these teachers retire, this element of the retraining problem is likely to diminish. Others served by these programs are teachers who are not authorized to teach secondary mathematics but otherwise hold a Standard, Single Subject or Multiple Subject credential.

District Certificate Programs. In San Diego County, three district initiated programs to encourage current teachers with weak mathematics backgrounds to upgrade their content knowledge.

Grossmont Union High School District has developed a cooperative training program for up to 20 teachers who will teach remedial mathematics. These volunteer teachers are being offered a choice of courses taught by other Secondary School teachers, some with college teaching experience. The six course offerings are listed below:

Basic Mathematics Concepts	Elementary Algebra
Intermediate Algebra	Plane & Solid Geometry
Trigonometry	Pre-Calculus

Participating teachers receive tuition-free credit through Grossmont Community College. The classes meet after school at one of the local High Schools.

For the past two years, the San Diego City School District has conducted a program in conjunction with Mesa College whereby teachers with degrees in other subjects earn the equivalent of a math Minor and become eligible for Supplemental Introductory Mathematics authorization under the California State Commission for Teacher Credentialing. A similar program is in effect for Supplementary English authorizations. Teachers take lower division mathematics courses at Mesa College, approved by the CTC, which earn them salary increments unless they are at the top of the salary scale. This arrangement is an exception to standard District policy in which lower division credits do not apply to salary increments. This serves as an incentive in lieu of paying tuition for participant teachers. Teachers may attend late afternoon or evening classes. Once the twenty semester units are completed, teachers are entitled to take the National Teacher's Examination if they choose, in order to qualify for a full Single Subject Credential. The

Mesa College Department Chair is available for counselling on a course of study and evaluating previous credits.

The course of study at Mesa College is aimed at those who will be teaching non-college track mathematics courses. The courses offered include:

Concepts in Elementary School Math	Intermediate Algebra
Trigonometry	Programming
Statistics	Plane Geometry
Math for Business Analysis	College Algebra & Trigonometry

It is interesting to note that the course of study does not include any teaching methods courses. This state of affairs exists because the State Credentialing Committee does not permit Education courses to count towards the credential. It is also interesting to note that San Diego State University was first approached by the District to offer a retraining program. However, SDSU faculty declined to participate reportedly due to the lack of agreement on course requirements. SDSU wanted to require at least one semester of calculus, while San Diego Unified School District felt this requirement was too stringent.

A North County consortium of school districts (Vista, Oceanside, San Marcos, Escondido, and Carlsbad) have developed, in cooperation with the SDSU Extension at San Marcos, a certificate program similar to the Grossmont Program. The program is aimed at teachers who do not have at least a Minor in mathematics, and at teachers already teaching mathematics who need refreshers, often Junior High Algebra teachers. Under the coordination of a SDSU CMP Institute participant, who has organized the program in her capacity as a Mentor Teacher, a scheduled two-year sequence of six courses from Basic Math to Pre-calculus is being offered. Twenty teachers are enrolled in the

first segment, to begin this summer. They represent Junior and Senior High teachers; inquiries from elementary teachers have been reported. Teachers have a choice of taking the courses tuition-free for no credit, or of paying tuition and earning credit which would count towards district salary increments. Federal support is being sought to cover tuition fees.

None of the programs require a course of study equivalent to a standard academic minor offered by the universities in the UC or CSU systems. Some of the required courses cover concepts generally introduced at the high school level and there is no course offering a college level treatment of calculus. No secondary mathematics methods courses are offered.

Identifying "Cross-over" Teachers. Based on the UCSD Teacher and District Surveys, we were able to estimate the number of teachers who require significant upgrading of mathematics content and instructional methods knowledge to teach mathematics effectively at the secondary school level. The term "cross-over" teacher is used here to mean teachers, otherwise fully credentialed, with limited backgrounds in mathematics, who transfer from other disciplines to teach mathematics. The following three characteristics have been selected as independent criteria for evaluating potential effectiveness as a secondary mathematics instructor:

Criteria	Explanation
A	Teacher has more than six years of mathematics teaching experience.
B	Teacher has at least a Minor in mathematics (both Junior and Senior High levels).
C	Junior High teacher has at least a Minor in mathematics and Senior High teacher has at least a mathematics Major.

Criterion (A) describes individuals who have been teaching mathematics for a number of years, regardless of their academic training in mathematics. Criteria (B) and (C) specify the amount of college mathematics a teacher has had. Criterion (C) is recommended as a minimum qualification for secondary teachers by the National Council of Teachers of Mathematics (NCTM).

Table 9 shows the percent of cross-over teachers failing to meet specified combinations of the four criteria. An estimate of the number of "cross-over" teachers resulting from each combination of criteria is also given (based on the 1982-83 CBEDS estimate 1265 mathematics teachers).

Table 9

Current Secondary Mathematics Teachers Not Meeting Selected Criteria (%)

	Criterion	Jr Hi	Sr Hi	Total	n (total)
1.	C	23%	50%	38%	430-531
2.	A or C	17	22	20	215-291
3.	B	23	16	19	202-278

NOTE: n (total) = projected number of cross-over teachers among the 1265 (CBEDS, 1982-83) mathematics teachers in San Diego and Imperial Counties (range computed based on 3-5% error)

Alternate Ways to Count Cross-over Teachers. The National Council of Teachers of Mathematics recommends, among other criteria, that Junior High mathematics teachers possess a minimum of an academic Minor in mathematics and at the Senior High level, a minimum of a mathematics Major. Based on the UCSD Teacher Survey, the first row in Table 9 indicates that as a result of applying criterion (C), between 430 and 531 local secondary teachers are in need of significant upgrading relative to their mathematics content knowledge.

Applying criterion (B), which specifies a minimum of an academic Minor for both Junior and Senior High mathematics teachers, gives an estimate of 202-278 current teachers who appear to need further mathematics training. It might also be argued that experience in teaching mathematics provides on-the-job training, which, while not ideal, should be included as a criterion when evaluating teacher effectiveness. We used an arbitrary minimum of 6 years mathematics teaching experience as reasonable on-the-job training. Using this criterion together with the NCTM requirements, (A or C), we projected that from 215-291 mathematics teachers are currently in need of mathematics content upgrading in the two counties. A two year program of study leading to an academic Minor and a Supplementary Credential authorization could be mounted by local Universities and Districts which would allow the effective upgrading of 50-75 teachers per year in time to do some good.

Table 10 shows the percent of secondary mathematics teachers responsible for at least one section of college preparatory mathematics (basic algebra and higher level courses). About 74% of the Senior High and 29% of the Junior High instructors teach algebra or higher level courses (Algebra +).

Table 10

Secondary Teachers Responsible for One or More Sections of College Preparatory and General Mathematics Courses According to the UCSD Teacher Survey (%)

	Class Type	
	College Prep	General and Career
UCSD Teacher Survey		
Jr Hi (n=323)	29.4	70.6
Sr Hi (n=381)	73.5	26.5
Total SD/Imperial (n=704)	53.3	46.7

About two-thirds of those Senior High teachers who teach basic algebra or higher level courses (n=280) reported having at least a Major in mathematics (only about 5% reported having less than a Minor). College preparatory classes are, in general, taught by individuals with greater content knowledge. The NCTM recommends that in particular, college preparatory courses should be taught by individuals with a Major in mathematics. Therefore, retraining teachers with limited mathematics backgrounds to a level equivalent to an academic Minor will not significantly increase the pool of teachers qualified to teach higher level courses. With the increase in demand for college preparatory courses, recruitment of fully qualified mathematics teachers is becoming critical. As the number of mathematics Majors diminish, larger and larger percentages of those remaining will be retiring or otherwise leaving secondary mathematics departments in the future. Instructional quality is likely to suffer at all levels.

Recent Mathematics Appointments. Over the past 6 years, 244 teachers (either new or transferred from within district) were appointed to teach

mathematics in San Diego and Imperial Counties. This represents about 34% of the current secondary mathematics teaching work-force.

Table 11 shows the appointment type distribution for all mathematics teachers compared with that for recent appointees. Whereas approximately 53% (21.1 + 32.2) of all mathematics teachers come from other mathematics departments (Transfer: Math-Dept) or began as fully qualified mathematics teachers (New with SS-Math - Single Subject Math Credential), only 29% (7.5 + 21.2) of those appointed in the last six years were appointed in these two ways. In fact over 70% (47.3 + 5.8 + 7.1 + 11.2) of all mathematics teachers appointed in the last six years were transferred from a non-mathematics department (Transfer: Non-Math) or came in under other irregular appointments.

Table 11

Secondary Mathematics Teachers of Each Appointment Type According to the UCSD Teacher Survey (%)

	Appointment Type					
	Transfer: Non-Math	Transfer: Math-Dept	New with SS-Math	New with Emergency	Temporary	Other
All Math Teachers (n=711)	29.5	21.1	32.2	3.2	4.2	9.6
Recent Math Appointees (n=241)	47.3	7.5	21.2	5.8	7.1	11.2

NOTE: Recent = 6 or less years mathematics teaching experience

Table 12 shows the percent of recent secondary Mathematics appointees reporting at least an academic Minor or Major in mathematics. Assuming that a mathematics Minor is the minimum content knowledge requirement for teaching

basic algebra, only 40% of the recent appointees would qualify. If a mathematics Major was required, only about 20% of the new-to-mathematics teachers would qualify. Since these figures are from a sample which may already be skewed towards more experienced teachers, the actual percent of qualified teachers under each condition may be somewhat lower.

At the Junior High level, a Major in mathematics is considered less critical since Junior High classes generally do not address content above basic algebra. At the Senior High level, where a Major in mathematics is the minimal standard recommended by the NCTM guidelines, approximately 75% of the recent secondary mathematics appointees did not meet this condition. About 60% did report having at least Minor in mathematics.

Table 12

Recently Appointed Secondary Mathematics Teachers With Less Than a Minor or Major in Mathematics According to the UCSD Teacher Survey (%)

	Less than a Minor	Less than a Major
Junior High (n=127)	42	87
Senior High (n=114)	40	75
Grades 7-12 (n=241)	41	81

NOTE: Recent = 6 or less years of mathematics teaching experience

Accordingly, Table 13 shows that most recent mathematics appointees at the Junior High level with less than a math Minor, report having a General Secondary or Elementary credential (55.7%). Most Senior High teachers with less than a math Minor reported having General Secondary or Other (administrative, designated subject, etc.) credentials (65.2%). This result

reflects the fact that a significant portion of the new-to-mathematics appointees come from the pool of experienced teachers transferred from other subject areas. Those recent Senior High appointees who had less than a Major in mathematics reported mainly General and Standard Secondary credentials, with some increase in the proportion of Emergency and Other authorizations.

These figures indicate a general lack of qualifications among this group as measured by type of authorization and mathematics background. The results point to the fact that many of the teachers with weak mathematics backgrounds have not acquired Supplementary Authorizations prior to receiving mathematics appointments. Overall, only about 3% of the recent appointees with less than a mathematics Minor have earned a Supplementary Mathematics Authorization.

Table 13

Credential Type of Recent Secondary Mathematics Appointees With Less Than a Minor or Major in Mathematics According to the UCSD Teacher Survey (%)

	Gen Sec	Stand Sec	Bd Res	Ryan Math	Ryan Supp	Mult Subj	Elem	Emerg	Other
Less than Minor									
Jr Hi (n=52)	26.9	5.8	5.8	1.9	1.9	7.7	28.8	5.8	15.4
Sr Hi (n=43)	41.9	7.0	9.3	0.0	4.7	0.0	0.0	14.0	23.3
Less than Major									
Jr Hi (n=107)	18.7	14.0	3.7	3.7	15.9	8.4	19.6	6.5	9.3
Sr Hi (n=84)	38.1	16.7	7.1	3.6	9.5	0.0	0.0	11.9	13.1

NOTE: Recent = 6 or less years of mathematics teaching experience

As shown in Table 14, about three-fourths of all Senior High mathematics teachers reported teaching at least one basic algebra or other higher level course (Alg +). Less than one-third of the Junior High teachers reported

teaching basic algebra. Among recent appointees, 46% of the Senior High and 16% of the Junior High teachers teach at least one basic algebra course (30% overall).

Table 14

Recent Appointees and All Secondary Teachers Responsible for College Preparatory Courses According to the UCSD Teacher Survey (%)

	Recent Appointees	Total
Jr Hi (Alg +)	16.1	29.9
Sr Hi (Alg +)	46.4	73.7
Grades 7-12 (Alg +)	30.3	53.7

Table 15 shows that of the recent appointees who teach basic algebra or higher level courses, 90% (50 + 40) of the Junior High teachers and 86% (37.3 + 49.0) of the Senior High teachers have at least a math Minor. Of these, 40% of the Junior High and 49% of the Senior High teachers have a Major in mathematics.

Table 15

Mathematics Background for Recent Junior and Senior High Appointees Teaching Algebra and Higher Level Courses According to the UCSD Teacher Survey (%)

	None	Minor Only	Major or MA
Jr Hi (Alg +) (n=20)	10.0	50.0	40.0
Sr Hi (Alg +) (n=51)	13.7	37.3	49.0
Grades 7-12 (Alg +) (n=71)	12.7	40.8	46.5

In summary, about 30% of the recently appointed mathematics teachers teach at least one basic algebra or higher level course. Of this group, the majority appear to have reasonably strong backgrounds in mathematics. It should be noted again that Minors are by self-report, and, that the category

(Alg +) includes all the college preparatory courses. Nearly 13% of new-to-mathematics teachers responsible for college preparatory courses have less than a Minor in mathematics. For all secondary teachers responsible for at least one college preparatory class, only about 5% have less than a Minor in mathematics. If this pattern continues, the quality of college preparatory mathematics instruction, along with the total mathematics program, is likely to suffer.

Continuing Education for Qualified Teachers. The previous discussion centers on the retraining needs of underqualified mathematics teachers. Though this problem is a serious one, it should not detract from the importance of providing appropriate continuing education opportunities for highly qualified teachers. It is vital to the health of mathematics programs that these teachers be encouraged to upgrade their teaching methodology and content knowledge.

Over the past 10 years, more emphasis has been placed on the role of problem solving in the mathematics curriculum. Advances in technology offer exciting new opportunities for exploring geometry, statistics, probability, algebra and calculus in the secondary school classroom.

The best qualified teachers must be encouraged to develop leadership skills to help less experienced faculty develop their mathematics content knowledge, develop new methods for integrating problem solving and computer applications into the existing curriculum, and design new courses to meet the needs of career and college bound students. The possible roles which the California Mathematics Project might play in developing professional leadership among the ranks of practicing teachers will be taken up at greater

length later in this report.

It must be noted, however, that any proposal to upgrade the mathematics content background of current teachers from other disciplines in order to increase the pool of qualified mathematics instructors must be regarded as a short term solution. Without an improvement in the status afforded those who choose teaching as a profession and other mechanisms which enhance job satisfaction, many of those trained to teach mathematics successfully are likely to be lost to higher status or more lucrative careers.

III. OBSERVATION COMPONENT

Overview

The UCSD California Mathematics Project conducted a follow-up study with several participating teachers on the effects of the summer CMP Institute run by SDSU and the County Department of Education. A detailed description of the activities, foci, and accomplishments of the summer program are described in the First Progress Report (CMP, 1983). The observational component of the current study began in October 1983. Its aims were four-fold:

a. to assess the influence of the SDSU CMP Summer Institute on the classroom practices of a sample of teachers.

b. to describe the job of the secondary school mathematics teacher in San Diego County.

c. to derive a description of mathematics practices common among the schools observed.

d. to use the results of the observations as a basis for assessing future teacher training needs in the San Diego area.

Monthly observations were conducted in the classrooms of six participants from the Summer Institute who were selected as a sample of the total group of participants. In addition to coded and narrative observation data, detailed interviews with the teachers were held in December and April of the 1983-84 school year.

The participants selected were teachers representing the following six categories:

- strong college mathematics background (Major) and (more than 10 years) experience teaching Senior High level mathematics
- strong college mathematics background and experience teaching Junior High level mathematics
- weak college mathematics background (less than a Minor) but with experience teaching Senior High level mathematics
- weak college mathematics background but with experience teaching Junior High level mathematics
- weak college mathematics background and was new to Senior High level mathematics teaching
- weak college mathematics background and was new to Junior High level mathematics teaching

The schools in the sample represented a cross-section of San Diego County schools except that schools serving a primarily Hispanic population were underrepresented. The schools ranged from those serving working-class to upper-middle class communities. One school was a Science and Math Magnet for the district and three were in urban areas.

Single visits were also made to the classrooms of several other Summer Institute participants. Three Saturday all-day workshops conducted by the Summer Institute staff were documented. At those sessions, questionnaires concerning various follow-up issues were collected from among the entire group of Summer Institute participants.

Material from the classroom observations, interviews, and questionnaires has been organized to address training needs for mathematics teachers along the lines of the Eight Recommendations for School Mathematics in the 80's, (National Council of Teachers of Mathematics, 1980). The framework of the NCTM Recommendations is consistent with the goals of the CMP Institute

organizers, and represents a thorough, delineated basis from which to assess the state of mathematics instruction in the local schools.

Classroom observations were conducted once a month for six months by L. Martin, Research Associate. One of the six targeted schools was only visited on two occasions since the participating teacher was temporarily removed from the classroom to do administrative work in the Magnet program. On each school visit, the observer most often attended three math classes, which may or may have represented three preparations.

A total of 76 observation hours were conducted. In January, observations were shortened to one class period for each site since it was felt that this revised schedule allowed valid sampling of certain activities of interest; however this did not prove to be the case, so a full schedule was resumed. During January 1984, Souviney, Co-principal Investigator, also observed in each classroom. At that time, meetings were held with the principals in each school in order to provide a progress report on observational activities, answer questions, and interview the administrators concerning their plans for future math instruction re-organization, inservice needs and staffing requirements.

In order to make the classroom observations systematic, a coding sheet was developed on which details of classroom structure, evaluation measures, and instructional contingencies were recorded. Listed below are the subcategories covered on the coding sheets:

Class Structure:	number of students classroom layout materials used lesson structure and sequence
Evaluation:	lesson objectives teacher's method for assessing students observer ratings of teacher effectiveness nature of student participation
Instruction:	methods of engaging students pedagogical techniques used to communicate content

In addition to the coding sheet, the observer wrote a running narrative of the lesson activities. This narrative included details of interactions and behavior, personal comments of the observer concerning lesson issues that seem to be relevant to the classroom activity, and more dense descriptions of lesson activities than were recorded on the check sheet.

Monthly summary observations for each teacher were compiled from the narrative "field notes" and from the coded information. In these summaries, questions and points to be followed-up were generated. The monthly teacher summaries provided a basis for questions addressed to each of the teacher participants in individual interviews conducted during the month of December. These interviews probed a number of general issues with each teacher and were also used to verify some of the assumptions and inferences made from the observation notes. The interviews were audiotaped and transcribed for further review. A general monthly summary was also prepared. This summery served the purpose of distilling themes that emerged during the site visits, and which are addressed in this report.

Problem-Solving as a Focus in the Classroom

The SDSU California Mathematics Project was unique in the State in two

respects. First, it included an evaluation component. Second, the Summer Institute was conducted jointly by San Diego State University and the Department of Education, San Diego County. Participants in the Institute represented two types of mathematics teachers from the San Diego and Imperial County region, (1) those with advanced mathematics backgrounds (A Option) and (2) those with less than a math Minor (B Option). Courses were offered appropriate to both groups of teachers. Both course Options of the CMP Summer Institute strongly emphasized problem-solving. For the experienced mathematics teachers in Option A, a program was planned whereby everyone participated in the problem-solving process, learned where to acquire materials for problem-solving activities, and discussed their experiences as problem-solvers. Dr. Ed Silver, the course instructor, stated his goals as follows:

> To provide creative experiences in problem-solving; to create common experiences in group situations to get participants to focus on their own problem-solving processes; to model alternative solutions; to transfer the creativity of the group to the individual students.

The new-to-mathematics teachers in Option B were presented with problem activities which illustrated conceptual issues being taught in their Math Topics class. For example, they calculated characteristics of an "average person" in the class, constructed a graph of the results and discovered these characteristics did not describe any one individual. This was accomplished in the context of a discussion on Measures of Central Tendency. Joan Akers, the instructor for the Option B Problem Solving sessions, stated that her goal was to develop an understanding that:

> Problem-solving designates real-world applications of mathematics in day-to-day situations, and in particular careers. It encompasses skills in conceptualization and notation, organizing and inference, and transfer and discovery

> of relationships among problem elements. Creative solution strategies such as visualization, using graphs and tables, looking for patterns, and estimating are fundamental as well.

The recent emphasis on problem-solving stems from a need to prepare students to succeed in a rapidly changing technological world and job market. The problem-solving activities to which CMP participants were introduced included the use of manipulative materials such as cards, beads, and string; visualization; graphs and tables; everyday sources of data such as newspapers. They comprised problems from all content areas from early Junior High topics to Number Theory.

Group Problem-Solving. Both Options of the CMP Summer Institute emphasized the use of small group work with classes. Some of the reasons offered for small group instruction were: (1) students get practice in articulating the steps used for reasoning, (2) they are more likely to come across alternate strategies, and (3), peers can serve as monitors to correct errors.

For the experienced teachers, the problem-solving course began with group work in which participants took different roles, monitoring and solving problems. Participants were invited to see how the presence of alternative viewpoints allows a small group to work with more material and to generate more possibilities than an individual. Gradually, the participants were encouraged to incorporate different roles in their own problem solving repertoire. By the end of the summer, each individual reported better problem-solving skills for approaching novel situations.

The teachers with less math background worked in groups of four throughout the summer. They were also shown that small group work serves

as both a motivator and a resource for students. Mixed ability groupings were encouraged and discussion took place about how students can be more creative in the relative "safety" of a group identity.

All participants in the Summer Institute became aware of the power of small group work as a result of the program; most projected that they would be using groups in their classes. In October, 64% of the teachers attending the Follow-up Session had tried group work. By January, the same number were organizing small-group activities on an occasional-to-regular basis. Option B participants used groups less, but raised the issue of group work for discussion more often than did Option A participants during Follow-up sessions.

Effects of the Summer Program. In a pre-summer survey taken among the teachers accepted into the program, no one mentioned acquiring problem-solving techniques or skills as a personal goal for their summer program. By the end of the first week of the summer program, problem-solving technique was, overall, the most frequently cited accomplishment participants projected achieving from the program. In an evaluation form distributed at the conclusion of the Summer Institute, the majority of participants of both Options felt that problem-solving had been very relevant to their needs and concerns as secondary school math teachers. Thus, the summer training raised awareness about problem-solving activities as a classroom focus for both experienced and inexperienced math teachers.

Was this awareness put into practice? In a questionnaire distributed in October of 1983, the first quarter of the new school year, 28 out of 36 respondents (78%) had introduced problem-solving activities into their

classes. By January, 50% of the people reporting (8 from Option A and 6 from Option B), said that they were doing problem-solving activities on a regular basis with their classes; another 13 people, or 46%, (7 from Option A and 6 from Option B) had such activities planned on an intermittent basis. One A Option participant was not using problem-solving in his room. At Follow-up Sessions, many teachers reported that they attended problem-solving workshops at the California Mathematics Council, Southern Section meetings and at the Greater San Diego Math Council meetings.

Observations in a sample of classrooms over a six month period showed several occurrences of problem-solving activities. Problem-solving lessons were observed in five of the six classrooms visited regularly. Three of the teachers gave their students the opportunity to earn extra credit by doing extra problems outside of assigned classwork. This is a new procedure for all teachers. The teacher working with remedial students does not give extra credit for problems however. He said that those who would do them are the best students anyway.

In all but one class, problem-solving bulletin boards were in evidence. These included colorful posters describing the Problem-of-the-Week, teacher-made problem displays, and lists of procedural steps to use when solving a problem. Teachers tended to use materials that had been introduced to them during the summer or at the regional professional meetings. (Note: Three of the teachers who were only observed on one occasion, organized problem-solving experiences during the observation time.)

In part, problem-solving activities were prompted by the visits from an outside observer, as were several other techniques promoted by the Institute

faculty. Teachers reported that they organized specific demonstration lessons and put up bulletin boards for the visits. The evaluators feel that this is a measure of the need and desirability for follow-up that is personalized, supportive, and professional (i.e., it often takes an interested audience to prompt performance). Since our teachers were practicing something new, they used the visitor as an occasion to motivate a change in their routine. One teacher chose not to, however. He instituted problem-solving for extra homework credit but did not put on a demonstration in class for the observer.

The teachers in our study had few colleagues who conducted problem-solving activities. They reported curiosity as well as skepticism among colleagues about what they were doing. Three of our six teachers were asked for problem-solving materials by colleagues.

All CMP participants contacted found problem-solving to be a very exciting innovation. They remarked on how the students at all levels perked up, paid attention, and persisted at the activity. Difficulties arose, however, in scheduling time to do problem-solving. Although one teacher at the October follow-up meeting reported that every Friday was devoted to problems, most other teachers felt that they could not take that much class time away from standard curriculum activities. Finding a flexible routine within which to institute a consistent series of problem-solving activities appears to be problematic. Some teachers held "problem-solving days" when they had extra class time or were caught up on regular work. Some gave students who finished classwork early a chance to solve problems. Others put problems on display for students to notice or ignore.

Ambivalence was noted towards having students work together at problem-

solving. Teachers reported, though, that groups worked better when the problems were more centrally related to the lesson material, rather than presenting groups with wholly new concepts or examples to work with. Sometimes a teacher put people in groups but instructed the faster students to not tell the answers too soon. Sometimes people worked together but got no credit for their effort, which was judged to be, from the students' vantage point, trivializing of the activity.

In general, teachers were unable to integrate problem-solving experiences into the standard curriculum. In part this seemed due to limitations in current textbooks and lack of systematic follow-up support provided by the instructors of the Institute. For both the A Option and B Option teachers, problem-solving per se was stressed along with group work during the summer, but fitting it into the routine was left an open issue. This is reflected in the wide range of practices observed during the school year and in the variety of successes experienced by the participants.

A tabulation of lesson-segment order in each of the classes regularly observed indicated that these teachers seldom deviated from a routine well-known to the students. The most frequent sequence was: a short drill—go over the previous day's homework—introduce new material--begin on the next day's homework. The last segment, starting on the assignment for the following day, was often a less structured, noisy time in classes. Teachers strongly expressed the need to include time for this activity since students had difficulty completing homework assignments without a "running start." It was the general consensus that none of the lesson segments could be successfully forfeited. Thus, we conclude, that if problem-solving is to be incorporated into instruction in the future, it must be carefully integrated into

textbooks and other district mandated materials.

A question was raised by the CMP Institute instructors concerning how much of the "true sense" of problem-solving can be incorporated into a prescribed course-of-study. Most felt that the essence of problem-solving is located in the teacher's attitude and cannot be written into problem-solving sections of a text. At the same time, changes in the CAP tests will mean that in the future all mathematics teachers will be required to include problem-solving as part of their regular curriculum. Given the observations above, it is important that an effort be made to establish useable guidelines for the integration of problem-solving into the secondary mathematics curriculum in a way that teachers can assess how successful they are at introducing these strategies to their students.

Teachers expended differing degrees of effort in integrating problem-solving activities into their lessons. Several methods were employed. Teachers gave problems for extra credit homework on weekends, rationalizing that students didn't do much homework on weekends anyway, or gave problem activities to the group when time permitted, generally about twice a month. One teacher gave a problem-of-the-week for extra credit, but found that some students tended to work on that instead of the assigned classwork. Other techniques included designing a formal step-by-step lesson in procedure, naming each problem-solving strategy as it occurred or requiring students to do problems as individual classwork, or presented problems to the whole group (sometimes dramatizing them) as a reward if everyone finished their assignments.

The two teachers observed who were new-to-mathematics instruction felt

that problem-solving required too much time-consuming organization. One of them, with four new preparations, felt he did not have time to plan as much as he would have liked. The other went about the activities quite formally, listing steps of procedures and exhorting the children to carry out various operations. She arranged a bulletin board with cues such as "Work Backwards" and "Do a simpler version" displayed.

The two experienced mathematics teachers with limited mathematics backgrounds were the most casual about including problem-solving in their lessons. They seemed to have a more limited view of what problem-solving involved and perhaps felt they were already emphasizing a problem-solving approach throughout the year. They did not seem anxious about whether or not they included specific problem-solving exercises in the curriculum.

The two teachers in our sample with strong mathematics preparation seemed to more easily integrate problem-solving into the teaching routine. Both, too, had open-ended approaches to solutions: they gave students partial credit for partial solutions and felt their role was to examine rather than divulge answers.

This characterization of the three types of teachers is somewhat simplified, yet generally reflects the differences observed. Similarities were also found among the teachers' overall interest in problem-solving. In one class, a competition developed between two sections when answers were posted as to which group could generate more solutions to the dollar-word problem (find words equal to $1.00 when the letters a-z are given the values 0.01 to 0.26). Competition was used on occasion by most of the teachers as a motivating device.

The Nature of Mathematics. Mathematics may be differentiated from many subject areas in that its mastery involves a high level of abstract thinking. Intermediate levels of mastery require less abstraction but are nonetheless constructed upon a type of induction which grants young thinkers more independence of thought than they experience in other subject areas. It is this key realization, according to the CMP participants, that is so hard for teachers to communicate to their students.

Both methodological and conceptual flexibility are at issue for the teacher and the student when problem-solving is introduced as a class topic. We mentioned the variety of ways such activities were incorporated into the class by teachers. All the teachers commented to the effect that problem-solving tackles the prevalent problem of students' conceptual rigidity. (This rigidity is one of the most frequently cited exasperations among experienced mathematics teachers.) The NCTM recommendations as well as the new California State Department of Education Mathematics Framework, seem to verify teachers' classroom experiences by emphasizing the need to provide flexible strategies to aid students in using mathematics to solve problems of interest to individuals and society.

With problem-solving being formally promoted at the State and Federal levels, teachers seem to feel new energy in their subject area. Problem-solving is challenging and fun for teachers as well. It presents the teacher with unpredictable responses from students and gives them a chance to engage in constructive dialogue in the classroom. There was the awareness among Institute participants that such an exchange can be motivating for students as well.

It is here, however, that problems may arise for teachers who are less confident of their skills in mathematics. Unpredictable situations may be threatening for them, and if so, may result in the teacher either maintaining instruction at a low conceptual level, or, structuring problem-solving lessons in a rigid manner. We have some evidence that this may be the case. It was also observed that less experienced teachers appeared more wary than experienced teachers of making errors in front of the class. Thus, teachers who will be new to math instruction should be made to feel as confident as possible in math content and in the process of mathematical thinking.

One of the teachers new-to-mathematics instruction was asked the reason why, when dividing fractions, you invert the divisor and multiply. The teacher replied that "we do this because that's the nature of mathematics," an inadequate response for someone responsible for the instruction of young minds. One goal of the summer Institute was to reduce the number of times such a response is given by equipping the new-to-mathematics teacher with sufficient conceptual understanding to field such unexpected, yet important questions.

Redefining Basic Skills

The NCTM recommends that the foundation of the mathematics curriculum must be expanded to reflect more than basic computation. In part, this recommendation is due to the ease of access to calculators and computers in the mathematics classroom. It is now increasingly possible to free class time for lessons in thinking skills and problem-solving by reducing the amount of mechanical practice of number and algebraic operations.

The CMP Summer Institute participants were encouraged to utilize calculators with their students, to teach estimation, induction, conjecture and proof, and problem-solving. Developing student awareness of the reasonableness of answers and checking work was also stressed.

Alternative skills were treated somewhat differently in the two sections of the Summer program. In the B Option, these topics were integrated with mathematics content (basic number properties, etc.) using a range of manipulative and other instructional materials. The A Option participants covered methodological issues and had a concentrated experience in using the computer as a tool for mathematical modelling and problem-solving.

Use of New Materials: Calculators and Computers. Observations in the classrooms of five teachers, over the course of six months, showed several trends in the use of alternate materials. No teacher in the sample used a computer with their class and none were in evidence in the classrooms. One teacher, who also taught a computer lab, introduced his students to Logo and other materials he had developed himself. He could not work out an effective way of having a machine in his room. Several teachers requested ideas for managing a class in which some students worked at a computer and others worked at their seats.

Final evaluations of the CMP Summer Institute showed that participants did not feel that they mastered techniques of incorporating the computer into everyday activity. The A Option teachers utilized computers as part of their course on "Applied Math Models" and participated in Logo demonstrations. These participants reported that the experience was valuable as a

demonstration of how computers could be used as a tool to understand aspects of mathematics which would be difficult to explore otherwise. For example, one teacher reported that using the computer gave him the opportunity to concentrate on the algebra instead of the mechanics of the problems; another reported using the computer to show applications of limits, graphing, and distance. Some participants expressed a desire to do more of this kind of activity. The Institute, however, did not seem to develop a new attitude among the teachers about computer use as it did in the case of calculator use.

All the participants were asked to summarize their involvement with computers in their mathematics teaching, both immediately after the Institute and during the school year. One teacher reported she learned a lot about computer use but since she didn't have one she was not able to use the information. At another school, all the computers were housed in a lab setting and were already scheduled for use in programming instruction and computer literacy. A few individuals made efforts to get the school to buy appropriate mathematics software or to change class scheduling to bring students into contact with computer activities appropriate for mathematics instruction. Among the six teachers observed during the year, none had approached PTAs or firms that had "adopted" their schools in order to initiate the purchase of calculators or computers.

There is some evidence from our interviews that computers are regarded as a separate domain of study by both teachers and administrators. Generally computer training for teachers was viewed as a prerequisite to learning how to teach a programming language. Because of this attitude, principals were understandably reluctant to shift mathematics teachers into computer

instruction classes. One teacher felt that since computer literacy in his district was being planned for grades 4-8, by the time he learned anything, the High School students would demand a higher level of specialization than he could acquire. Another teacher, with good computer skills, plans to seek some computer class assignments next year, but is not particularly hopeful.

It seems reasonable to say that some emphasis in teacher education programs should be placed on the differences between being able to teach computer programming and being able to utilize preprogrammed software in the mathematics curriculum. The differences seem to be overlooked by those planning and implementing mathematics programs, and may account for the relative slowness with which this innovation is taken up in math classes.

All six observed teachers allowed their students to use calculators. The most regular usage was found in the remedial High School classes. B Option teachers, in general, used calculators more than A Option teachers with their students. However, the way in which they were used varied more with the individual teaching styles than by Option. In two schools serving lower income students, teachers had boxes of calculators to pass out as needed, whereas in other schools students had their own. In one High School Career Math class, a student comment seemed to exemplify often heard concerns regarding the use of calculators, "it's not a test if we can use a calculator." Students also asked why it wasn't cheating to use a calculator and if teachers allowed students to use them in college. One teacher pointed out that while a calculator can help you solve a problem, it won't solve the problem for you.

Other New Materials. There is evidence that teachers started the year

attempting to implement some of the new materials they learned about in the summer, reduced the number of these activities in mid-Winter, and later resumed their use in a more selective way. This material included problem-solving activities, visuals and manipulatives, group problem-solving activities, and estimation worksheets. For the CMP group as a whole, nearly 90% said that they had changed their planning for the year. However, at least 25% reported on a lack of appropriate materials available in schools. Many were concerned about the time required to prepare such materials.

Topical bulletin boards, mentioned earlier, tended to go up early in the year and stay up, unchanged. One teacher in our sample reorganized and added to his displays over the Christmas break. He also displayed student work on his bulletin boards, which seemed to motivate the students to produce more work for display. It seems that bulletin boards serve different purposes, but their real effect may be unknown. For instance, several teachers had posters too small to be read from the seats. Do students mill around and read them before and after class? Some had posters, particularly the mnemonic type, which students were encouraged to refer to when completing assignments, but often did not. If a display changes too often, is this distracting to students? One teacher found that if he wrote a problem-of-the-week on the board at the beginning of the hour, students worked on that instead of classwork. Presumably, the extensive walls of a classroom are a potential educational aid, yet their use is guided intuitively for the most part.

In the case of overhead projectors, which were the only audio-visual media in use besides the chalkboard and worksheets, use is more clearly dictated by function. Teachers who used this aid generally found it simplified their lessons with students.

The two new-to-math teachers observed had two and four math preparations, respectively. In addition to preparations for other classes taught, there were district objectives, proficiency tests, and textbooks to cover. Most of the new-to-math teachers expressed great unease about deviating from the text, and were unsure what topics could be skipped or rearranged. One of them remarked in an interview that every time he takes a professional course and decides to try innovations, it means "problems to straighten out" which are not always worth resolving. He cited the "license to socialize" taken by the students when he introduced small-group problem-solving as the reason he cut back on such work. This teacher did, however, improvise on textbook use with his classes. The more experienced teachers in our sample worked with similar constraints but were clearly not as controlled by the instructional materials. Rather, they treated the text as an additional resource upon which to draw.

Although there was support for teachers' attempts to innovate from within the immediate school community (Department Chair or Principal), coordination and time for new and relevant lesson development was lacking. As mentioned above, teachers felt that introducing problem-solving would take time away from regular lessons. For new-to-mathematics teachers, integrating new materials may be daunting. In fact, one of the teachers spent all her free time preparing and giving students early-morning catch up time in order to include problem-solving and manipulative experiences during the regular math period.

New Techniques. A number of teaching practices appeared to change as a result of the Summer Institute. One teacher, who has been teaching High

School mathematics (and Algebra at the Community College level) for more than 10 years, but who had little college mathematics background, reported that he did not change his methods very much as a result of the Institute. Still, he and the experienced Junior High teacher with a strong mathematics background, both introduced problem-solving for extra credit as a regular part of their program. The three other teachers, all with limited college mathematics background, conducted problem-solving activities on an irregular basis, in some cases for extra credit. The experienced High School teacher with a strong mathematics background, introduced problem-solving as a class time activity for those students who had finished some other assignments.

According to our observations, teachers with less math teaching experience tended to oversee innovative activities more than those who had more math teaching experience. For example, those who felt less acquainted with the problem-solving process themselves, tended to make it a public activity under their control.

Another dimension on which teachers were assessed was the method used to instruct students. All the teachers but one reported a new effort to use hands-on materials and visual displays in getting key concepts across to students. The one who did not use manipulatives frequently used visual displays on the chalkboard.

All of the teachers observed emphasized estimation processes, although to varying extents. In at least two cases, this was not a new emphasis but the teachers seemed to derive confirmation of their approach from the summer course. Four teachers created bulletin board displays with algorithms and reminders of solution steps for various problem types. One new-to-math

teacher said that his displays of scientific notation helped him and his students develop problem-solving strategies since it saved figuring and checking time when working with large numbers. One teacher observed that since his textbook did not cover estimation, he had neglected it. Another learned during the summer that estimation was a good way to help students develop "number sense". All the teachers reported that instead of estimating, students most often did an accurate calculation and then rounded off.

All the teachers were also observed to explicitly emphasize awareness of the operations in use during mathematical activity. Such emphasizing of operations was stressed in the summer program. Some teachers pointed out operations by "warning" students that a particular operation was needed to solve a problem or identified certain "tricks" the students could employ to identify the operation required (e.g., "of" means to multiply). Depending on the context, emphasizing operations was judged to vary in its usefulness to students. This is an example of where the attempt to be flexible in instruction, reinforced by the summer program, interacted with varying degrees of mathematical knowledge and with teaching styles.

In all classes observed during large group lessons, most calculations were done on the board by the teacher. Partial solutions were solicited from the students as they worked. Teachers responsible for a range of course levels showed less complete board calculations with their higher level classes, often skipping steps in the algorithms. For the teachers new-to-math instruction, this was slightly less in evidence, although over the course of the year there was an increased incidence of previously introduced mathematical shortcuts observed.

The new-to-math teachers seemed to use the chalk board as an organizational device for their lessons. In some cases for the new-to-math teachers, the detailed working out of problems on the board, or the more step-by-step explications they felt comfortable with, worked to the advantage of the class. The teacher's need for clarity often assured that all the operations were clearly revealed.

The three Junior High teachers and the remedial High School teacher all remarked on the weakness of their students' number concepts. Three of these teachers conducted regular drills on number facts. Although decreased emphasis on this type of activity is called for by the NCTM, all the teachers felt they could not do an efficient job unless the students were continually practicing and consolidating the number facts.

All the teachers observed utilized examples from everyday life to illustrate the lessons, again, with varying degrees of aptness. Everyday applications were introduced at all levels of instruction, except in the most advanced math classes observed (Intermediate Algebra). Teachers often re-framed textbook word problems. For example, they converted the estimation of the speed of trains to the more common problem faced by Air Traffic Controllers in figuring the intersection of airplane routes. In several cases, selection of examples appeared to be a deliberate instructional device for getting students to relate mathematics to common situations.

One teacher always introduced a concept with a group discussion by raising his own set of problems before introducing the topic in the textbook. He felt the book might confuse students by not directly addressing the students' questions. Another teacher observed that he was planning several

different approaches to presenting topics as a result of seeing the variety of explanations others came up with during the Institute. He also cautioned that too many different explanations can sometimes overwhelm the students. The students in his class, interestingly, also tended to come up with very original questions about the material.

Institute participants, particularly those teaching Senior High courses, expressed the need for courses dealing with secondary mathematics content they were expected to teach. Participants remarked that they missed the opportunity to share with their colleagues, effective, field-tested techniques for communicating specific mathematics topics. The organizers of the Summer program have taken this need into account in planning for next years Institute. Methods classes will be grouped by grade level or course type taught, rather than by participants' mathematics backgrounds as was the case in the 1983-84 CMP Institute.

The Use of Mathematical Language. The secondary school students in the classes observed were not given much responsibility for expressing mathematical relationships or describing organizing aspects of problem solutions. Although each teacher asked students to name operations, to label variables and operators, and to name the next step in the exercises, most often it was the teacher who set up the initial equation and formulated the next questions in the sequence. Two teachers in our sample had similar classroom routines which did require students to provide a sequence of steps at particular times in the lesson.

The phenomenon whereby teachers did more modelling of mathematical talk than shaping of students' talk is judged to be due primarily to the

achievement level of the classes observed and the nature of the material. All the teachers, with and without strong mathematics backgrounds, did at some time ask students to say how they went about setting up and solving a problem. At the same time, the teacher was most likely to re-frame the student's response, clarifying it and shaping it into a succinct statement. In the Advanced Algebra/Trigonometry classes observed, the students more often offered an approach with the teacher extending the logic of the proposed solution. By the time students reach the advanced levels of mathematics, they have gained increased mastery in communicating mathematically.

One would then predict that individual teachers would vary these techniques according to the level of the class they were teaching. A count of questioning techniques utilized during the various lessons observed over the year showed that the tendency to use shaping procedures (see Mathematics as Subject Matter) varied between teachers (i.e., some tended to use these more than others overall). However, all teachers used these more in the higher level classes they were teaching. Thus, the frequency with which teachers explicitly encourage students to generate and articulate mathematical communications can be represented by a bell-shaped curve -- less at the lowest and highest levels and most in the middle level classes.

Related to this issue is the notion of insistence on technical vocabulary. Articulating procedures and using mathematical terms may not co-vary. For example, in one class the teacher had a rigorous routine for getting students to describe their exercise set-ups, yet common language terms and short-cut mnemonics were emphasized at the same time. Insistence on correct terminology was observed intermittently in each classroom observed.

However, the terms seemed to be used with fluency by the two teachers with stronger math backgrounds and by the Senior High teacher with little formal background in mathematics but extensive experience teaching mathematics at this level. It may be that, with increased experience, this aspect of mathematical language usage becomes familiar and presumably more readily available as a model for the students.

The CMP Institute provided teachers with an opportunity to analyze the content and instructional methods for secondary level mathematics instruction by reconstructing the High School learning experience. This integration was exemplified in the mathematics content courses by the simultaneous introduction of mathematical problem-solving activities and demonstrations of problem-solving approaches to teaching. It may have been more difficult to extract a direct model of how to present material in the classroom from the shifting framework of being both student and analyst for new-to-mathematics teachers. Two of the experienced mathematics teachers in our sample, who had little formal college mathematics, showed the least change in routines and instructional methods as a result of the summer program. Both however, incorporated many activities introduced during the Institute and felt reinforced in their current practices. They also tended to rely on familiar routines. Both the new-to-mathematics teachers and those with stronger mathematics backgrounds teaching higher level courses appeared most flexible in adopting new techniques and instructional methods (though with differing degrees of fluency), the former because of inexperience and the latter because of conceptual flexibility.

Applying Stringent Standards

As discussed above, the teachers observed tended to follow routines for segmenting class time, which were rearranged only for special activities. These special activities did not occur often and were sometimes organized because the observer was present. To the extent that interested, supportive classroom visitors have this effect, visits should be encouraged. This is one feature of the paired-teacher program suggested by many of the CMP participants as being helpful, primarily the model of more and less experienced teachers visiting each others' rooms.

Teachers can by no means be characterized as inflexible about rearranging and improvising schedules; but, the responsiveness that enhances teaching efficiency, being to a large extent a function of preparation time and class size, is not easily effected.

Rearrangements of routine in basic level math courses seemed to involve large units of reteaching activity. For example, a teacher wanted to introduce as a new topic the concept of ratio and was frustrated because students had forgotten how to cross-multiply fractions. The result was a reorganization of the lesson to reteach and practice the prerequisite skill. In an Advanced Algebra course, sidetracking often entailed suggesting an alternate solution path -- a less drastic change of course. It seems that those dealing at the beginning levels of secondary mathematics instruction would be required to be the most skillful in determining the "cost-effectiveness" of rearranging particular activities. We can expect, then, that new-to-math teachers would need to focus on this issue to a greater extent in their pre-and in-service training.

During the Institute, B Option participants were introduced to a

conceptual view of mathematics content and given training in detecting the underlying misunderstandings of students. However, scheduling and use of class time was addressed in a less formal way. Overall, the message was clear -- priority should not be given to isolated drill of algorithmic skills; the use of manipulatives for concept development should be encouraged; and the interrelationships among mathematics topics should be highlighted.

The A Option participants, those with more mathematics background, addressed the same issues, but with less actual practice with new materials. While participants expressed some desire to have more hands-on activities, observations and conversations during the year showed that these teachers were better able to generate their own materials and to schedule class time for innovative exercises.

Classroom management, including student management, was not dealt with in the Summer program as a separate topic. This was perhaps due to limitations on time since the Institute instructors were fully engaged in presenting subject matter in innovative ways. Classroom management was, however, cited by teachers before and after the summer, as the most important issue next to the learning of mathematics content. It should be noted that the need for such mundane training may be central to the training of effective basic mathematics instructors. Observations in classrooms during the school year raised several issues related to efficiency and effectiveness.

Specific Motivation. Teachers recognized that repeating basic skills ad nauseum is drudgery for all concerned. This was especially noted by the teacher responsible for remedial and career math classes. Drills are

particularly problematic when a class shows a range of ability, as most do, and may exacerbate behavior problems. In contrast, offering more complex problems which were relatively easy to solve excited both the able and less able students. Certainly, the novelty of problem-solving held the interest of students in the classes observed but was not evident when long exercise sheets were distributed.

Each teacher used different sets of motivators to "change the attitude" of students who were unenthusiastic or fearful about mathematics. The methods seemed to interact with the school population served. Since a generally positive attitude on the part of students was viewed by most teachers as the appropriate starting point for a teacher, the motivating aspect of effective teaching strategies emerges as highly significant.

Socio-Economic Factors. Each of the teachers observed expressed an individual philosophy and attitude towards their job. However, observations of these teachers' practices show an interaction between the needs of the particular community served and the teacher's articulation of a general teaching theory. The teachers have been shaped by their students.

For example, the teacher who taught a number of large remedial classes in a lower-middle class school that bussed in students from poorer neighborhoods, attributed many of the learning problems to the students themselves (e.g., poor preparation at earlier grade levels). He also cites as problems the drug busts, the fights, and the salient social concerns of the students which overwhelm them when they come into a class. When they are asked to work in groups, they quickly become unmanageable. Group work was eliminated largely for this reason early in the school year, and was

continued only occasionally in the Algebra I class. Although a good deal of class time was devoted to the next day's homework assignment, which seemed an inefficient use of class time, the students reportedly did not do homework otherwise. The teacher indicated that the threat of a poor grade was often ineffective with these students, yet negative reasons to do math tend to prevail in these classes.

The High School teacher in an upwardly-mobile, high income suburb also found that small group work gave students a license to socialize and subsequently cut back on the use of groups. He also allowed time for students to work on homework in class. His students did their homework routinely and were quiet during lesson time, although classes were very large. This teacher planned lesson segments around his students' tolerance for listening (about 15 minutes) and how close they were to vacation or lunchtime. Although he said his students need more review of elementary operations than he would like, he assumes an ability to learn on their part and his own ability to teach given his ability to arrange external circumstances.

In a Junior High in that same geographic area, a new-to-math teacher working with remedial students devoted a lot of time to enhancing the students' self-image. In the first few months of school, she talked to them about making errors, about how they may be victims of ineffective teaching, and how she would not let them think of themselves as failures. Over the year she cut back on the exhortations and reinforcements she used and placed more demands on students, but continued to provide before and after school tutoring time. Thus, a humanistic-psychology approach dictated some of the selection and organization of mathematics activities. In this setting, rewards and punishments were seen as a concern to parents who were involved

in their childrens schooling primarily through phone calls, notes, and conferences.

Another Junior High teacher in a community of slightly lower income also emphasizes positive experiences for his students. As he felt that these students were not as likely to do homework and to stay on-task as students from a higher income community, he had evolved a very entertaining approach to maintaining motivation. He felt it was important for him to change his presentation of materials often, involve students actively, and use humor in the large classes he teaches. The philosophy guiding class organization is, "they're good kids but they can't be trusted yet."

The third Junior High in our study was located in a wealthy section with a significant number of students bussed in from lower income neighborhoods. The character of the school as a whole seemed unclear to us. In the math classes we observed at this school, the teacher operated under the constraints he felt from his students -- including their self-perception of academic success. While the children from different neighborhoods have been learning to socialize, there was a problem in planning activities to meet everyone's needs. This teacher regularly utilized many innovations from the summer program which, in fact, seemed to serve as a common ground for his students. There was a sense in the classroom that if students behaved, they wouldn't be punished and might even have fun.

Establishing and Shifting Innovations. Over the course of the year teachers tended to institute changes and stick with them for some domains, and to shift emphases and organization in other domains. As mentioned above, at least two teachers started out using small group work with Senior High

students and then discontinued it soon after, only employing small group work on a limited basis with the highest level classes.

One new-to-mathematics Junior High teacher recognized that her tolerance for noise and talking was low. She did not introduce groups into her class until February, and this was as part of a documented project she was doing for a Master's program. This teacher was very systematic about preparing her classes. In spite of her sensitivity to the noise level, she felt introducing groups would have a good chance of being successful since all aspects were thought out beforehand. She maintained a good deal of control over the groups. For example, she told certain students not to give away answers too soon. She generally put aside her own reservations for the sake of following the project through. Later, she reduced the use of groups because she felt the students did not listen to her when working together.

Most teachers we observed tried to implement some new materials during the first third of the year. They returned to more traditional materials by mid-year. Cyclical changes in implementation of new materials and other innovations may reflect external influences on the class routine outside the teachers' control. For example, upcoming proficiency tests seem to be correlated with an increase in drills; the end of a grading period may allow teachers to bring in new materials to try out, since grades have already been recorded; the beginning of a new semester means new students, class size changes and confusion.

Observers rated the "informativeness" of the teachers, as an estimation of how well a teacher explained material. These observations showed increased effectiveness through the Fall, and a slight downward trend in the early

Spring. There was also some fluctuation related to topics. In general teachers showed increasing clarity and effectiveness in the first few months that they then maintained over the course of the year.

Teachers showed shifts in the messages they were communicating to students about mathematical thinking and motivation. On the whole, the school year started with teachers providing support to students by discussing some of the difficulties encountered when learning mathematics. Later in the year, the emphasis seemed to shift. The students were given more responsibility for knowing or the material and teachers withdrew some of the emotional and technical props (e.g., writing out solutions on the board). One teacher noted that her students were using a more logical approach on their own towards the end of the year (even if they didn't arrive at correct answers).

One measure of this trend is how explicitly teachers stated the lesson goals for the day. Although there were individual differences between teachers, there was a trend later in the year for the content goal of the lessons to become implicit rather than explicitly mentioned. In some cases, this effect held for the upper level classes, while lower levels continued to need explicitly stated objectives. The statement of lesson goals is considered by the evaluators to itself be a lesson in mathematical thinking.

There are implications to be drawn from this effect for introducing innovation. Given a tendency of student-teacher relations to become routine, teachers will need periodic encouragement to reflect on the expectations they communicate to students. It seemed from observation and written reports that the three Saturday Follow-up sessions of the CMP Summer Institute did at least in part, serve this function. Teachers spent time discussing their own

responses to innovations and the effects of the innovations on students, and analyzing the next steps to be taken with their classes.

Constraints on Efficiency. Before and after the CMP Summer Institute, the most frequently cited concern teachers had about being effective was the influence of large class size. As one of the CMP teachers recently put it, "class size is the bottom line."

Certainly the class sizes of 28 to 35 students (for 5 periods per day) appeared unwieldy. The crowding, the lack of contact with the teacher, the amount of time each student had to perform, impaired efficient and effective organization of class time. It was easy to see how attempting more participatory or small group problem-solving experiences would be undermined in such conditions.

Teachers also said that the range of ability among students in large classes encourages teachers to aim instruction at a level which serves the needs of very few. In the advanced level classes and in the small remedial classes, this problem did not appear. For a beginning mathematics instructor, the demands of teaching effectively under such conditions are great. Any attempts to minimize teaching to the "common denominator," such as within-class subgrouping or on mixed-ability small-group instruction, demands a strong support system and sophisticated classroom management skills. Most secondary teachers have little previous training or experience with these methods.

Mathematics as a Subject Matter. Studies of learning processes indicate that the best way to insure that students have assimilated new

material is by having their responses shaped by the instructor. That is, the learner has some natural tendency to respond which is used as a starting point and brought to resemble the desired goal response through a feedback process. The units of a response being shaped and the frequency or strength of the corrective feedback are not fixed, which is why the intuition and experience of the teacher is vital. In contrast, responses which learners are called upon to perform because of an external criterion (e.g., a high test score), without having any partial previous knowledge, may be acquired by learners but do not necessarily serve to reliably guide individuals' responses in new contexts (Reynolds, 1975). Modelling desired response is a technique midway in effectiveness between rule learning and shaped learning.

One of the criteria we employed in judging the effectiveness of instructor methods was how much shaping they did in contrast to eliciting the repetition of new chunks of material or unelaborated statements. It turned out that in the classrooms we observed, there is very little shaping going on, although there was more in the classes of some teachers. These findings are discussed in a later section of this report.

Much of the Junior High and early Senior High levels of mathematics instruction seem to place constraints on the possible kinds of instructional dialogue. Truly alternative methods of instruction were not observed on a regular basis, and it is hard to envision what such methods would be given the kinds of constraints mentioned above.

Most often, shaping occurred in the context of problem-solving. For example, one teacher handed out problem sheets to his Algebra I class. The interaction sequence, which is an example of where the teacher's notion of the

problem determines the shaping exchange, was as follows:

1. Teacher calls on someone to say what the problem is asking for.
2. Teacher points out how this problem is different from a similar one the class has had.
3. Teacher models a set-up on the board.
4. Teacher asks the students to provide appropriate values.
5. Teacher describes the first solution step.
6. Teacher solicits the next steps from students until the problem is solved.
7. Teacher states the meaning of the equation again and gets them to say what is different about it from they're previous experience.
8. Teacher demonstrates how to check the solution.

Small group work is probably the clearest example of peer shaping. Written work can be shaping as well; it is what repeated drills accomplish. One teacher reported on the pride her students take in seeing how much faster and more accurate they become on written multiplication drills and how they notice their responses transfer. Using everyday examples can also serve to shape students more effectively in the use of mathematical material. In another class, students were seen to pay close attention when the teacher used their names in rate problems about people driving cars. Another teacher called his students to the front of the room to act out problem scenarios for the class to solve. Manipulatives and multiple approaches to solving the same problem can have a similar effect. For example, one teacher drew pictures of geometric figures on the board and also displayed cut-out figures to give students visual cues for generating descriptions of the properties of the different shapes. As students used their own words to respond, the teacher recast their answers using more technical terms, such as "vertex" and "congruence."

As effective as these techniques are, they are still subordinated to large group work. The prevalent type of large group work is a lesson led by

the teacher who serves as a model, working out problems on the board, and eliciting small fill-in comments from students. It seems that inferential thinking, a goal of a mathematics program, is most often modelled for students rather than developed using a shaping process. This finding suggests that more emphasis on math labs and on decentralized methods in the classroom is called for in re-training programs.

Evaluation Using a Wide Range of Measures

Measures for the evaluation of Math Programs at the secondary school level are outlined in a publication of the NCTM (1981). Evaluation of a successful mathematics program at the individual class level most often depends on unit and cumulative tests. Since tests have pre-determined criteria for success and may not have been written by the teachers using the instrument, other types of measures are now being called for in the assessment of class progress and school effectiveness. Achievement tests often do not inform teachers about students' grasp of curriculum materials. Because test scores have been misused for tracking students, putting ceilings on standards, and comparing school districts, mathematics educators are wary of depending on standard achievement scores for meaningful program evaluation.

The NCTM recommendations are themselves unspecific about alternative measures to tests. However, they are specific about activities and materials which should be included in mathematics programs. While they recognize that measures of teaching competency are often of questionable validity, at the same time they encourage an intuitive search for measures of student competency.

Discussions with Institute instructors prior to the Institute showed that even they were unable or unwilling to specify "mathematics proficiency" or to describe what it was they hoped to see change in their students. Inferential thinking skills, "mathematical thinking" and a tendency to identify multiple approaches to a problem, were listed as desirable characteristics but were not clearly definable.

Problem-solving, estimating, and common sense are not easily evaluated at the pre-college level. A further problem in assessing programs, as expressed by several Institute participants, is that the effects of good teaching may not show up until years later. Educators continue to disagree about naming specific criteria to evaluate learning at the secondary school level. While all the participants came to accept calculators as a means to do away with time-consuming operations, they also continued to complain about students' lack of preparation and understanding of basic number facts. All teachers were aware of the limitations of "teaching to the test," however they indicated that their departments generally took time to review and drill for the district-and state-wide tests.

There was less of a tendency to talk about students' preparedness among teachers of older students. Instead they talked about students' abilities. The reason of such teachers centers on experience. If a student has been taught fractions three or four times and still hasn't mastered them, teaching quality is no longer the issue. Such reasoning is faulty especially in domains where teachers lack effective instructional techniques. Socio-economic differences seem to account for much of the stratification of participation in mathematics programs within and between schools.

The Instructor of problem-solving for the A Option teachers gave pre- and post-course problems to solve and evaluated the flexibility expressed in the participants' answers. Participants were also asked to assess their own performance. For most participants in this course, problems were reported to have become somewhat easier to solve, although the instructor planned for a constant level of difficulty. Assignments in their problem-solving course were rated at a constant level of difficulty by the B Option teachers. In part, this may be explained by the differences in concepts underlying the problems. The differences in perceived assignment difficulty between the groups is interesting in that the instructors judged both problem-solving courses to be successful, although participants clearly learned different skills.

At the conclusion of the summer, participants were asked to project the most likely changes in their practices during the coming year. Two of the five most frequently cited predictions had to do with alternative evaluations: being more aware of the teacher's role, and, encouraging a discovery learning process in students. Participants in each group also mentioned evaluative techniques they had covered in their respective methods courses, such as having students keep problem notebooks and revising homework procedures. The projected use of the Mathematics Diagnostic Testing Program instruments was also indicated by many participants.

From our questionnaires and conversations during the summer, there was some indication that teachers did not learn new diagnostic techniques or new ways to set performance criteria in their classrooms. Many had mentioned these as materials they wanted to learn.

In general, the instructors of the Summer Institute modelled an activity-oriented approach with informal evaluation of teaching effectiveness. This model arose from the faculty's desire to maintain a voluntary, non-punitive atmosphere and from an attitude, seen among mathematicians, that changes in thinking are in some ways unmeasurable.

Local Evaluation. San Diego County comprises 15 secondary school districts. These districts variously set standards for their secondary school students. In one affluent district observed, a full set of sequenced curriculum materials has been developed for teachers and minimum proficiency tests are administered. All districts administer the California Assessment Program (CAP) tests to assess school program effectiveness.

For the most part, however, teachers are left on their own with local school community support for establishing evaluation measures. Communication between a department chair or principal and the staff varied in effectiveness among the schools observed. In some cases, District Guidance Counselors may override a teacher's recommendation about promoting a student to the next level of mathematics. Teachers in at least two schools complained that new students may be shunted from class to class for several weeks until the proper placement level is determined.

The strength of the local professional community is important for the teachers in gaining useful feedback for their work. Most CMP participants would like it further strengthened through team or paired-teaching for inexperienced mathematics teachers, within-school leadership activities, and through local in-service programs.

The goals stated by the teachers in our study and among the CMP participants as a whole, emphasized influencing student attitudes more than influencing the development of specific mathematics skills. Conquering fear, accepting tests as a challenge, mastering the content material, gaining confidence yet being realistic about skills, were some of the lessons teachers hoped to impart. Evidently, student enjoyment is a central measure of effectiveness to teachers, both because their job is made more rewarding and because they know that a positive attitude is more likely to result in learning.

It is not clear, though, how the atmosphere created in a classroom can or should be evaluated. Nor is it clear to what extent test scores are an indirect measure of classroom atmosphere. Until motivational or attitudinal measures can be better defined, exams will continue to be used as assessments of individual ability and their results used to judge students rather than programs.

Observed Assessment Measures. The measures of student performance used by teachers in our observation study were as follows:

Table 16

Measures of Student Performance

	Teacher					
	#1	#2	#3	#4	#5	#6
Daily Drills			x			
Other Drills/Quizzes	x			x	x	x
Graded Homework	x	x	x	x	x	x
Unit Tests	x	x	x	x	x	
Final Tests						x
Seatwork Problems		x				
Extra Credit Assnmts	x	x		x		x
Games						
Small Group work						

Informal measures of students' comprehension were observed as well, such as calling on particular students, probing answers for further clarification, and asking for further questions students may have had. Teachers seemed to be tracking the students' learning to facilitate long range progress in their programs. Teachers used such input to decide when to review material.

When teachers were asked how they evaluated their own performance, most initially replied that they get a "feeling" they are communicating and that the students are learning. They use student restlessness as an indicator of having lost their audience. They also cited unit test scores as evidence of student achievement.

From our sampling it was not possible to judge the relation that might exist between feedback the teacher is receiving (such as, wrong answers to questions) and a sense of having communicated more or less effectively. It was possible, however, to observe teachers' changes of strategy in the face of student confusion. All the teachers observed were judged to be generally responsive, that is, able to shift techniques and approaches to material when the need arose. Some teachers demonstrated more variability in this regard

though it did not seem break down by mathematics background or grade level taught. In certain instances, the effectiveness of the shifts did seem to favor those with greater experience teaching mathematics.

In-class observations yielded information about how often teachers explicitly stated the goals of the lesson. Such a practice was judged to be useful in helping students organize their actions appropriately. Differences among the teachers in our study on this dimension related to personal teaching style rather than to years of mathematics teaching experience or level of class taught. Two teachers consistently articulated lesson goals to their students and the others, 40-70% of the time.

Use of MDTP Instruments. The Mathematics Diagnostic Testing Program (MDTP) instruments (see Appendix C) seemed to be useful tools for teachers of college-bound students. These instruments were designed to provide specific feedback to students, parents and teachers regarding individual preparedness to advance to higher level courses. MDTP test results can also be used to identify individual areas of weaknesses so that appropriate corrective measures can be taken. The Instruments do not purport to test every aspect of course subject matter, but rather to assess those concepts which have proven to be predictive of success in subsequent mathematics courses. Students, parents, and teachers have used the results of these tests to inform their school careers and lesson planning. There is less tendency to use these tests for comparisons too since they are not tied to State funding and do not constitute part of a student's grade. Many Institute teachers encourage the development of a Pre-Algebra level test. Further information on MDTP is given in Section IV of this report.

Effects of the Program. Participants in the CMP Summer Institute became acquainted with a variety of activities to use in their classes which demanded new kinds of performance from their students. The summer faculty attempted to sensitize participant teachers to a range of ways to demonstrate concepts and to analyze the conceptual thinking of their students.

Expanding Range and Quantity of Mathematics Offerings

During the 1983-84 academic year, eight of the sixteen High School Districts in San Diego County raised their minimum mathematics requirements for graduation. Five of the eight raised requirements from one to two years. The remaining, including the two largest districts, raised requirements from two to three years, effective in 1987. There are now no districts requiring less than two years of High School mathematics for graduation.

The CMP Summer Institute participants were asked to indicate if there were plans in their respective schools to increase the course offerings in the coming year. Of 31 respondents, six reported new courses will be offered and three reported the possibility of this happening. The new courses included:

Remedial Math using computers; Problem-Solving (possibly)
Computer Science
Computers for 8th Grade
Probability and Statistics (possibly)
Calculus (possibly)
Computer Literacy
Basic Algebra 1-2
Math III-IV (non-college prep)
Applied Math

Class size averaged 28 students in the regular classes we observed and

17 students in the remedial classes. In one school, an Algebra section was dropped midyear so enrollment in the Algebra class we observed rose to 39 in January. At the same time, our surveys showed that class size as a determinate of effective teaching was a prime concern to the majority of teachers in the CMP group. The noble goal of raising mathematics graduation requirements should not be addressed by simply placing additional students in already overcrowded classrooms. As one new-to-math teacher put it:

> "I think if I had 18 or 20 students and I had two preparations at the most [he has four] ... I would really ... develop an individualized program. I can't do it with 33 ... Personally, I want to teach the highest level math classes I can. Why should I fight with kids ... and spend my time walking around the room getting them to sit down, keep their books open, trying to get them to read the problems that are assigned ... When you have a class that's low in numbers you can contend with that, but with a class of 30, it's pretty hard."

The concept of "laterality" of choice, that is, offering students several different career paths to algebra, advanced mathematics or applied topics, does not exist in the schools we visited. In some sense, it can be said to exist in "career" and remedial courses in which ninth, tenth, and eleventh grade offerings often repeat previously presented material.

Several problems may exist in changing to a truly lateral system. In articulating concerns they had about math classes, teachers, especially those teaching remedial students, expressed concern about student failure to grasp ideas and about the difficulty of communicating concepts. Secondary mathematics programs are usually designed in terms instructional objectives. There is an assumed hierarchy in the subject matter sequence as it is defined. Arranging the curriculum to allow alternative paths for entry to advanced mathematics topics will require a rethinking of these underlying hierarchical assumptions. The current course-of-study effectively limits

access to advanced courses to those students who are committed to a college preparatory program as entering freshmen.

It is interesting to note that recent studies of everyday problem-solving show that practical mathematics competence is one area where college-bound and non-college bound students look very much alike (Rogoff & Lave, 1984; Rosin, 1973). A change in the design of the secondary curriculum to attempt to address subject matter in all courses using a problem-solving approach might be one way to improve the potential choices for students wishing to extend their mathematics development beyond the minimum required.

Among the schools we visited, there were no reports of unmet needs among gifted students. Some districts have expressed concern with the underrepresentation of minorities in Gifted and Talented Education (GATE) programs. Those students identified for the GATE program should continue to be relatively well served since the majority of teachers with the most advanced mathematics backgrounds were eager to teach these students in every school. It is possible, however, that these students are not being maximally extended at some schools since the needs of poorer students may overwhelm the attention of the staff. The lessons observed in GATE classrooms, all conducted by teachers with strong mathematics backgrounds and at least eight years experience, may be described as exemplary, both from the point of view of teacher proficiency and of student participation. Our observations were limited to only a few sites, however, so generalization to the total school population would be inappropriate.

New Directions in Mathematics Curriculum. The focus of the CMP Summer Institute was on improving already existing math sequences by the inclusion

of problem-solving activities, manipulative materials, and emphasis on real-world applications. The value of certain less frequently emphasized topics, such as Probability and Statistics, was dealt with as well. Specific course syllabii and textbook materials were not analyzed specifically in the summer program. Participants expressed their desire to have more detailed discussions on these topics, a suggestion that has been incorporated into the plans for next summer's CMP Institute.

Discussions of the leadership role of experienced teachers, likewise, did not focus on planning alternative curricula. It may be that the format of the training program by necessity stresses the individual teacher and decisions in the classroom. The Summer Institute did introduce participants to the EQUALS project, whose work is aimed at developing programs to encourage women and minorities in math and science. A representative of EQUALS gave two afternoon workshops to Option A and B participants. The reviews of the workshop were very favorable on the whole. Participants mentioned in particular that the EQUALS materials answered many questions on the important issue of how to deal with mixed motivational levels within the classroom. At the January Follow-Up sessions, about one-third of the 29 teachers attending reported that they had used EQUALS materials. Forty percent of the teachers attending the final Follow-Up session arranged for students at their schools to attend the "Expanding Your Horizons" conference sponsored by EQUALS, held on a Saturday at San Diego State University.

In contrast to the 28 per cent who tried EQUALS materials with their classes, 97 percent had used some problem-solving activities and 79 per cent had used small group work. We conclude from this, that the likelihood of change in the classroom occurring from training programs is more a function

of the amount of time spent with new materials in training sessions, than of how inherently interesting or innovative the new materials might be.

Flexibility in Using Materials. The CMP Summer program introduced participants to new materials and resources for use in the classroom. Some discussion occurred about textbooks and some participants said they would have profited from more specific textbook review or procedures for comparison. This is not an implausible situation since, out of six introductory algebra classes observed in six schools, only three different texts were in use.

In one urban High School visited, the whole faculty reviews potential textbooks. The CMP teacher at this school, with three years of mathematics teaching experience and twenty-six total years experience, felt relatively bound to follow textbooks he did not personally like since the entire faculty was involved in making the selection. Another teacher, in a suburban High School, with limited mathematics background and fourteen years of mathematics teaching out of twenty-two total years teaching experience, felt very comfortable not using the standard textbook, improvising lessons, and switching topic sequences. Similarly, the Junior High teacher with limited mathematics background and fourteen years experience mathematics teaching experience, stopped using the text he was using at mid-year because he and his team-teacher planned to use another book in the second semester. In another Junior High, the entire district used the same basic algebra book and publishes guides that expand on topics presented in the textbook. The experienced teacher observed at this school, who had a strong mathematics background, supplements the required lessons with material from texts he finds in the library. The other teacher in our sample with limited experience

teaching mathematics, used the fairly structured district materials, and her own supplementary exercises.

In summary, teachers in different districts work under different constraints about curriculum guidelines, but those with more experience find ways of using textbooks more flexibly. This suggests that training programs for teachers should include instruction on effective ways to utilize textbooks as instructional tools in an effort to promote flexibility in class activities to meet diverse student needs.

Encouraging Professionalism

Effective teachers clearly must be up to date with developments in their field of instruction. In the case of mathematics, this has come to mean familiarization with new instructional practices (e.g., problem-solving techniques) as well as upgrading and refreshing of mathematics content knowledge. The UCSD Teacher Survey revealed, however, that only about one-fifth of the teachers in the two counties have taken more than one class in methods since they began teaching, in contrast to nearly half who reported taking more than one mathematics content course.

Participating in the CMP Institute. The motivations for teachers to apply to the CMP program ranged from the need to refresh skills and feel more secure about concepts to the stipend. For most teachers, the $600 summer stipend represented a sacrifice from regular summer employment (outside the school system) but one that, in the final analysis, they were glad to have made because of what they accomplished. On the final questionnaire, all but two individuals felt that they benefitted professionally more from the

Institute than other workshops or in-service programs they had experienced.

Participants stated that they were led to consider mathematics content, applications, curriculum, and pedagogy to a greater extent than they had previously in other training situations. They felt the course material to be extremely pertinent to their professional needs for the most part and found their interest in the subject stimulated. When participants were asked to generate a list of what they expected to see change in their classes as a result of their summer experience, some of the more frequent responses were judged to reflect a change in professional attitude. For example, "being aware of the teacher's role" and "acting as a resource for other teachers." The majority of applicants reported plans to maintain contact with other participants. Strong participation in the three Follow-up Sessions helped many stay in touch with each other throughout the year. Considerable energy and creativity was expressed by the group and the spirit of comradery developed throughout the year. A measure of the regard held by the summer faculty for the teachers was the seriousness with which they considered the feedback of the participants. The major concerns and suggestions generated by the A and B Options were addressed in the program revisions for the 1984 Summer program.

Leadership Training. The SDSU CMP Institute indirectly promoted leadership activities among A Option participants by offering individual encouragement and information on opportunities for involvement as resources within their departments. The B Option participants were also encouraged to act as resources for fellow teachers. At the October Follow-Up session, four of the 36 teachers responding indicated that they had already given workshops at their schools; about half the teachers said they had informally shared

information with colleagues. In January, the Follow-Up Session was devoted to discussion of issues of retraining and models of teacher support systems. The results of the discussions were summarized in a memo written by the D. McLeod, Project Director and circulated among the staff and participants.

At the time of this writing, participants from the summer program have accomplished the following leadership activities:

2 have led workshops at the Regional NCTM meetings
2 participated in a panel at a MDTP Conference
1 has been put in charge of a magnet school mathematics program
3 have been hired as teacher-leaders for the 1984 Summer Institute
1 is coordinating a district consortium's program for upgrading teacher mathematics knowledge through SDSU North County Extension
2 have become involved in a County curriculum development project

Although the 1983 program encouraged professional leadership among participants, the teachers did not judge that specific leadership skills were emphasized in their courses. The 1984 CMP Institute Program includes a specific leadership training component in which three selected teachers will lead the Summer Institute seminars, receive training in the development of in-service programs, and be introduced to various County resources. Teachers chosen for this job will themselves participate in a week-long training session before the Institute begins, and will be paid a salary.

Ironically, when the initial proposal for the 1984 CMP Project was submitted, the Project Advisory Committee questioned the amount of money to be paid to the teacher-leaders: "The payment of $1000 to returning teachers seems excessive in light of what has been said about their contribution to the project." The project leader pointed out in his reply that, as the proposal indicated, these leaders would be working for five weeks, one of

which would be devoted to training, and would be responsible for a daily seminar. In contrast, University level faculty will be receiving between $3,000 and $4,600 for their participation.

We feel the attitude expressed by the Advisory Committee is itself indicative of the non-professional status accorded to Secondary Educators, and is particularly detrimental to the efforts of groups attempting to seriously deal with the retraining situation. Every major report concerned with improving the school situation cites poor economic incentives as a major contributing factor to the crisis. One major goal of the SDSU project is to assist teachers to serve as resources for each other. For 1984, the number of teacher-leaders was cut from the proposed six teachers to three.

Other Influences on Professionalism. While the Summer Institute was judged to be a successful first step in promoting professional attitudes, teachers' jobs are affected by more than personal development. In questionnaires distributed before the summer session began, participants were asked to rate their level of job satisfaction. The results showed that, on graded scale from 1 to 5, with 5 being the highest, the mode response for all participants was 4. The mean response for the A Option was 3.04 and 4.0 for the B Option.

In Follow-Up questionnaires, job satisfaction did not show an increase, possibly because ratings were high initially. Many participants reported a decrease in job satisfaction at the same time that they reported a higher level of professional interest in work. Since we know that the summer program did have an effect on classroom practices, according to the teachers' reports, we may infer that overall job satisfaction does not depend on having

access to progressive instructional models, although a sense of professionalism may.

Classroom control and behavior management was a topic which many teachers expressed an interest in, but which was not directly covered in the Summer program. Suggestions were given for checking homework, or for planning math games for fun, but basic management techniques were not addressed. Sixty per cent of the participants felt that such a workshop would be desirable at a follow-up session.

The smaller and more middle class rooms were quieter according to our observations. One of the teachers in our sample who taught remedial Junior High math to 18 students in a middle class district, felt her students had problems of decorum. She chose to begin the year with an emphasis on the children´s feelings of failure and fear of failure. She constantly praised any efforts at working out problems, responding, and taking chances, and planned successful experiences for them. She reminded the students that successes are not automatic for adults and that trying is how you learn. In this atmosphere, students and their parents reported improvements in attitude; the classes were working a large proportion of class time and completing assignments. Later in the year, these supports were required to a lesser degree.

Job satisfaction and a feeling of professionalism come in large measure from how much control teachers feel they have over important decisions in their work. There were varying degrees of this among the classes we observed. Several teachers did not know their teaching assignments until school started in September, and one had his schedule changed two months into the

year. Three of our six teachers did their classroom planning independent of the school administration and department chair. One was isolated from the department members but worked closely with the principal; another worked with the department chair but not the administration; and the sixth served in an administrative role herself.

Several teachers expressed discouragement at being at the mercy of administrative decisions. One cannot plan career goals if the school may be eliminating certain courses or cannot guarantee a schedule three months ahead of time. One teacher, in her district six years, was still being interviewed almost yearly for a permanent position within the district. In May 1984, this teacher was informed that she has been given tenure in the district but has lost her position at her school, which went to an administrator who wanted to return to the classroom. She has since secured a position in another school, teaching computers and one math section. Some of these problems, of course, are the results of constraints the schools are under. However, they highlight the fact that educational change will be hard to effect at a local level without broader change to give stability to programs.

Districts vary widely in their policies for in-service support. Some organize their own, some support teachers to get training in local institutions or to attend conferences. Most schools we visited had very limited resources allocated for inservice training. It was more often the case that only one or two teachers in a school were permitted to attend a professional conference such as the Greater San Diego Mathematics Council Annual Conference or the California Mathematics Council Meeting.

When CMP participants were asked to choose their preferred model for

career training, given that they had 12 month appointments, most still preferred a Summer Institute model. Secondly, people indicated they'd like a choice of different ways to acquire further professional training. Week-end and evening workshops were least preferable. The CMP teachers clearly preferred on-site professional support from colleagues to other alternatives. The least preferred model was the District or TEC Center drop-in center offering workshops and consultants.

Four of our teachers were involved in regular cooperative exchanges of instructional materials with colleagues. They all found this an important feature of their work. One new-to-mathematics teacher found being a resource for colleagues professionally gratifying. As previously mentioned, several participants held informal demonstrations or workshops at their schools. Others were asked to give workshops at local and regional professional meetings. On the first Follow-Up questionnaire in October only six out of 36 participants reported that their principals had inquired about their experience at the Institute. This kind of activity was encouraged by the Summer faculty, and should increase in the coming year as the leadership component of the program develops. Being able to work cooperatively with colleagues seemed to be a strong motivational factor during the Institute and when participants returned to their schools.

The Electronic Message System. As an attempt to extend collegial contact throughout the year, the five regularly observed teachers from the Summer program, their instructors, and other CMP staff were linked by an electronic mail system. The plan was to hook-up all the parties on microcomputers through the phone lines as a convenient means to initiate discussion and share ideas. The system allowed individuals to communicate

privately or address messages to groups.

The hope was to document the conversation among participants and to develop a model that could be instituted on a broader basis. Several difficulties prevented the message system from being used extensively:

1. Although modems were acquired for each of the participating sites and the computers were compatible with available software, persistent technical difficulties occurred with connections to the school telephone system. Several schools took months to go through district channels to have an extension installed near the computer. In one school, the outside lines had to be reached through the school operator; since the central computer handling the messages often "hung up" when the connection was delayed by the operator, the school switchboard often had to be redialed. Other technical problems with the phones, both in the schools and in the long-distance lines, proved daunting. All the sites were finally on-line in February 1984.

2. In some cases, the computers were located in places that were hard to reach for the teachers. Teachers were not very likely to walk to the school library (in two cases) to use the machine; in one school, the computer was located in another teacher's classroom.In the end, modems were installed in two of the teachers' homes. In two schools, other teachers were involved in networking projects through the University and so they shared access with the CMP teachers.

3. The training we provided for the teachers in the use of the message system proved to be inadequate for teachers to feel comfortable approaching the machines on their own. A demonstration of the electronic message system

was given during the first Follow-Up Saturday in October for the six teachers. Subsequently, when the modems were installed, each teacher was given a demonstration using their machine and given written documentation. On a couple of occasions, the observer went over the procedures again with a teacher. One teacher, on his own, asked a former student of his to assist him with an extra tutorial (the delayed connections interfered with his motivation, however).

In the end, some mail was exchanged between all the teachers and Project staff. Again, when contact was made, teachers were clearly excited and eager to utilize the medium. Organizational problems prevented this kind of contact from being reliably established. It is felt that if the technical problems are overcome, this medium of communication among teachers, university faculty, Department of Education personnel and TEC Center consultants, would be valuable.

The Two Options. The San Diego CMP Summer Institute is unique in the State since it included a program component, funded by the Department of Education, San Diego County, directed at teachers with limited backgrounds in mathematics. This cooperative arrangement was felt by the organizers to add strength to the program's abilities to meet local training needs. Indeed, the mix of backgrounds proved to be important during the Institute in fostering professionalism among the participants.

In spite of spending most of the day in separate courses, a dialogue was initiate between the two groups of teachers. This was specifically the focus of one Saturday Follow-Up session, in which small groups composed of A and B Option participants discussed support systems for newly assigned mathematics

teachers. The problems that exist in the Junior/Senior High transition were also discussed openly, and opinions were aired. Because of the diversity of experiences represented by this group, the Institute faculty was able to generate a frank discussion of current problems in mathematics education and possible solutions.

The effectiveness of this dialogue assured the planners of future Institutes that promoting such contacts between teachers would be invaluable an feature to include. The plans for the 1984 CMP Institute include three Options to accommodate an even wider range of mathematics backgrounds and a teacher-leader program. The three Options are designed for teachers with strong mathematics background, usually a Major, teachers with some college math, probably less than a Minor, and teachers with very little college mathematics. The curriculum for each option will include a strong problem-solving component and participants will re-group for methods classes according to their teaching interests. The evaluators estimate that this plan will contribute greatly to the overall aim of advancing professionalism among the mathematics instructors involved.

Public Support for Mathematics Instruction

Besides bringing together both experienced and non-experienced secondary mathematics instructors, the SDSU CMP represents a unique collaborative effort among educational institutions in the San Diego community. The two major universities, and the Department of Education, San Diego County and several local Districts provided resources and personnel for the Institute and Follow-up Sessions. Within SDSU and UCSD, individuals within the Mathematics and Education Departments made contributions to the success of

the effort. Representatives from the TEC Center and the Greater San Diego Mathematics Council (GSDMC) served on the UCSD Advisory Committee and assisted with survey dissemination.

The course of study offered to teachers during the summer of 1983 did not directly address community organizing issues, although these issues came up in discussion. A message emerged that teachers would have to become active professionally in their districts and in local professional organizations in order to encourage changes necessary to improve the quality of mathematics education in San Diego and Imperial Counties. Many participants wondered if they could be successful in influencing curriculum decisions and the local School Board. Several participants have become active in the local GSDMC and NCTM activities because of the Summer Institute. One of the participants works actively in the CFT.

Many teachers expressed a wish to see more parent education. The strongest sentiment in this regard concerned obtaining parental support for children to finish homework assignments and to see the need for learning mathematics. It is not clear how this could be achieved, however, since reinforcing of school values in the home seems to be more strongly related to socio-economic factors rather than to "attitude" alone. It may be that the school itself will have to take more of a responsibility for finding incentives for students and to structure mathematics learning as a meaningful activity.

The schools observed in our sample represented a cross-section of neighborhoods and a range of parent support systems. In all, the primary contact teachers had with parents was associated with discipline problems.

One teacher commented that in his school, parent conferences, potentially a positive influence on student achievement, were consistently scheduled too early in the school year to provide parents with meaningful information.

In one upper-middle class suburb, the Junior High classroom teacher had parents organized to grade papers and construct materials. In another middle class area that bussed in low SES students, the local parent organization purchased supplies for the mathematics department. Another middle class Junior High was "adopted" by had a large electronics firm. The firm represents a potential resource for the school. In the two schools serving primarily minority populations, including the Mathematics and Science Magnet School, parent participation was not apparent. For parents of students in college-bound programs, the MDTP instruments may offer an opportunity for effective involvement in their schools' mathematics programs.

As of this writing, the San Diego TEC Center has not decided upon the specific use of their resources in the area of improved mathematics instruction. The TECC Policy Board has been meeting to evaluate various alternative means of service available to it. They are currently collecting information about District in-service requirements and plans, and are awaiting the results of this Project in an effort prior to making plans for the 1984-85 school year.

IV. MATHEMATICS DIAGNOSTIC TESTING PROJECT

In order to gain further understanding of teacher training needs in San Diego and Imperial Counties, an analysis was planned of the results of the Mathematics Diagnostic Tests administered to area students in the Spring of 1984. The Department of Education, San Diego County assisted with the notification of individual schools about the availability of the MDTP exams. The exams and scoring service were provided free to teachers with the support provided the CSU and UC Systems. A. Manaster (Co-PI) gave a presentation to the SDSU Institute participants on the development of the MDTP Exams and the procedures for using them with their classes. A MDTP/CMP conference was held in the Spring, 1984 to familiarize secondary teachers with the Exams and discuss administration and scoring procedures. A follow-up meeting was held early in June at SDSU for teachers who had given the Exams.

Due to the late return of completed exams from the schools (many arriving in mid-June), a complete analysis of the results was just underway at the time of this writing. A total of about 8000 San Diego and Imperial County exams were scored this year by the Office of Learning Resources at UCSD. This represents an approximately 400% increase in usage over last year. Of the 25 public schools participating in the MDTP this year, nearly half were sites where CMP Institute participants were currently teaching. Approximately one-half of the 8000 MDTP Exams returned were Basic Algebra, about 30% were Intermediate Algebra, and the remaining were comprised of the two versions of the Pre-calculus exam (see Appendix C for descriptions of the MDTP Exams).

An analysis of the demographic characteristics of the students taking the exams as well as a description of the achievement profiles for various subgroups is currently underway. For further information on the results, please contact Dr. Alfred Manaster, Professor of Mathematics, University of California, San Diego, La Jolla, CA 92093.

V. CONCLUSION

The activities of the joint UCSD/SDSU California Mathematics Project which developed and analyzed the District and Teacher surveys, and delivered feedback to Institute organizers concerning the training program was unique among the CMP projects statewide. The fact that this Project, which did not provide direct services to teachers, was seen to offer an important contribution to our understanding of the needs within the mathematics education community, is a measure of the commitment CMP is making to the long term improvement of mathematics instruction in the State of California. The Project offered us the challenge to build cooperation among institutions of Higher Education, local Districts, the TEC Center and the Department of Education, San Diego County. We are grateful for the consistent help we have received from everyone involved in the Project. Dr. Douglas McLeod, the SDSU CMP Institute Director, was particularly responsive to the comments and observations we were able to make regarding the Summer Institute.

We measure our contribution first by the changes that were made in the SDSU Institute for the coming year. Many innovations resulted from information culled from our evaluations. The information we collected also helped to verify the effectiveness of certain program components. The activities of the observers during the summer contributed to the faculty's reflections on program planning and to teacher-participants' consciousness of professional development. During the school year, the regular presence of an observer in five classrooms was seen to serve as a support for teachers who were trying to improve their classroom effectiveness. This role could be

assumed in the future by CMP Teacher Leaders or by locally identified Mentor Teachers. Experience gained from our attempts to develop an electronic message network among participants should help future efforts of the kind.

The survey information and analysis will be valuable in planning for the area's future mathematics needs. This kind of large scale effort has been shown to be feasible given the generally cooperative efforts of District Personnel officers. The results will be available for the future inservice planning at the District and County levels. Those districts which cooperated with us on the District Personnel survey will be sent the results for their respective districts.

Indications from teachers are that the results of the MDTP exams will be used by teachers to assess the sequencing of their instructional program. Students and parents can use the results to help plan future course work and can share the results with with counselors who can help them plan their programs. Communication with school counselors is cited as an important factor in class composition which, in turn, affects teaching capabilities.

We hope the results of this study prove useful in support of the long term effort to improve the quality of mathematics education in the State of California.

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APPENCICES

APPENDIX A - The UCSD District Survey

APPENDIX B - The UCSD Teacher Survey

APPENDIX C - Mathematics Diagnostic Testing Project Exams

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TEACHER EDUCATION PROGRAM (D-002)

LA JOLLA, CALIFORNIA 92093
TELEPHONE: (714) 452-2957

September 22, 1983

TO: DISTRICT PERSONNEL DIRECTORS

SUBJECT: Survey of Professional Backgrounds of Mathematics Teachers

The attached survey of professional backgrounds of mathematics teachers (grades 7-12) is an important component of a study commissioned by the California Mathematics Project. We are attempting to document the status of professional qualifications and teaching experience to help predict future preservice and retraining requirements for mathematics teachers.

As you may remember, this survey was forwarded to your office by Mr. Joe Bullington, Assistant Superintendent, Personnel Division last June. Due to the narrow response "window", returns were less than expected. I would greatly appreciate any effort on the part of your office to provide the requested information. A summary report will be forwarded to your office upon completion of the study.

Special Instructions: Please provide information relative to the 1983-84 school year except as noted. If data from 1983-84 is unavailable, include the most recent figures and note effective dates.

To save time, I am mailing this direct to your office. You may also receive a letter from Mr. Bullington's office regarding this matter. Please return the completed survey to:

Randall Souviney, Academic Administrator
Teacher Education Program D-002
University of California, San Diego
La Jolla, CA 92093

Thank you for your assistance with this matter.

Sincerely,

Randall Souviney

Randall Souviney
Academic Administrator

encl.: Survey

CALIFORNIA MATHEMATICS PROJECT
Survey of Professional Backgrounds of Secondary
Mathematics Teachers (Grades 7-12)
(for School Year 1983-1984)

Personnel Officer _____ District _____

Title _____ Telephone _____

Definition: For the purposes of this survey, a secondary mathematics teacher is any individual who is assigned to teach as part of the standard program of instruction a minimum of one mathematics course (or a minimum of one mathematics class per day) in grades 7-12. Please include teachers that were not considered an integral part of the mathematics instructional program but are teaching standard classes (e.g., math-lab teacher). We are interested in gathering data for the 1983-84 school year except where noted. If unavailable, please use most recent data available and note effective date.

District Information

1. Enter the number of schools in your district by category (include each school in only one category):

   a. Comprehensive High Schools (9-12 or 10-12) _____

   b. Continuation High Schools _____

   c. Atypical High Schools (must include 10-12) _____

   d. Junior High Schools (7-8 or 7-9) _____

   e. Middle Schools (must include 7 and 8) _____

   Total Secondary Schools _____

2. Total enrollment for 1983-84 by grade level:

Grade	7	8	9	10	11	12
Enrollment						

3. Total number of mathematics sections for:

1981-1982

	First Semester	Second Semester
Grades 7-8 (or 7-9)		
Grades 9-12 (or 10-12)		

1982-1983

	First Semester	Second Semester
Grades 7-8 (or 7-9)		
Grades 9-12 (or 10-12		

1983-1984

	First Semester	Second Semester (Projected)
Grades 7-8 (or 7-9)		
Grades 9-12 (or 10-12)		

4. How many of the new math sections added for the Fall of 1983 (over 1982) were covered by:

	1983-1984 First Semester
a. Teacher transfered within school	
b. Teacher transfered within district	
c. New contract teacher (with appropriate credential)	
d. New contract teacher (with emergency credential)	

Comments: _____

_____

_____

_____

5. Years of mathematics required for high school graduation in 1982-83: ____

   Years of mathematics required for hish school graduation in 1983-84: ____

6. Please attach a copy of the Spring semester 1982-83 master mathematics schedule for each school (including the names of instructors) for each school (grades 7-12).

7. Please enter student enrollment for each class on the master schedule(s).

TEACHER INFORMATION

SCHOOL _____

Authorizing Credentials: (see Column 1 below)
a. General Secondary
b. Standard Secondary (Major or Minor)
c. Annual Board Resolution
d. Single Subject (Mathematics)
e. Single Subject (with additional Math Authorization)
f. Multiple Subject (with Supl. Math Aurhorization)
g. Emergency
h. Elementary
i. Other
j. Unknown

Appointment Type: (See Column 2 below)
S - Internal Transfer Within School
D - Transfer Within District
C - New Contract - appropriate credential
E - New Contract - emergency credential
N/A - Not Applicable

Teacher Name	Authorizing Credential (a-g above)	New 1983 Appointment only Enter S, D, C, E, above	Highest Math Degree a. Minor b. Major c. M.A. d. Ph.D. e. None f. Unknown	Year of Math Degree	Year of Birth	Year of Employment in Present District	Total Years Teaching Experience
0. J. Doe	b	C	a	1971	1948	1982	13
1.							
2.							
3.							
4.							
5.							
6.							
7.							
8.							
9.							
10.							
11. 115							116

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TEACHER EDUCATION PROGRAM (D-002)

LA JOLLA, CALIFORNIA 92093
TELEPHONE: (714) 452-2957

January 15, 1984

To: Mathematics Teachers, Grades 7-12

California Mathematics Project Survey:
Professional Background and Teaching Assignment

The attached survey is part of an extensive study of secondary mathematics teaching, funded by the State Department of Education and the California Mathematics Project. Its purpose is to help local school districts, universities and the Teacher Education and Computer Center (TECC) establish efficient retraining and recruitment efforts for mathematics teachers. Specifically, the results of this survey are needed in order to accurately predict:

1. The number of "crossover" teachers from other disciplines which are likely to require retraining in mathematics.

2. The types of inservice courses needed to assure quality mathematics instruction during this period of increasing graduation requirements.

If you teach at least one mathematics class per day in grades 7-12, please complete the attached survey. We regret asking you to respond to yet another survey but this one really is important--for you, your colleagues and the profession. Your response will be anonymous and the results will not be reported by individual school or district.

As in indication of our appreciation, if at least 60% of the mathematics teachers from your school return the completed surveys, the school will be included in a random drawing for a copy of "Graphing Equations", a software package for Apple II microcomputers. Thank you for your assistance.

Please return the completed survey to your Department Chair/Math Contact Teacher.

Randall Souviney

Randall Souviney
Principal Investigator
California Mathematics Project

RS:dmt
Enc.

California Mathematics Project
Mathematics Teacher Survey

School District.....

San Diego County .... Imperial County .... Form # .....

		Column #1	#2
1. In column 1, please check the lowest grade-level to which you currently teach mathematics.	(grade 7)	--|--	
	(grade 8)	--|--	
	(grade 9)	--|--	
2. In column 2, please check the highest grade-level to which you currently teach mathematics.	(grade 10)	--|--	
	(grade 11)	--|--	
	(grade 12)	--|--	
3. In column 1, please check the total number of years you have been teaching.	1--3	--|--	
	4--6	--|--	
	7--9	--|--	
4. In column 2, please check the total number of years you have been teaching mathematics for one or more periods per day.	10-12	--|--	
	13-15	--|--	
	16-18	--|--	
	19-21	--|--	
	22-24	--|--	
	25-27	--|--	
	28+	--|--	
5. In column 1, please check the total number of semester college units of mathematics content you have completed since employment as a teacher (do not count computer courses).	0--6	--|--	
	7-12	--|--	
	13-18	--|--	
	19-24	--|--	
6. In column 2, please check the total number of semester college units of mathematics teaching-methods you have completed since employment as a teacher.	25+	--|--	
7. In column 1, please check your highest earned degree in mathematics.	a) Minor (academic)	--|--	
	b) Major (BS or BA)	--|--	
	c) MA	--|--	
8. In column 2, please check your highest earned degree in any subject area.	d) PhD	--|--	

9. Please check the type of credential you have, authorizing you to teach mathematics.

a) General Secondary (or Junior High)	--
b) Standard Secondary (Fisher--with math major or minor)	--
c) Any credential with Annual Board Resolution	--
d) Single Subject (Ryan--mathematics)	--
e) Single Subject (Ryan--with supplemental math authorization)	--
f) Multiple Subject (Ryan--with supplemental math authorization)	--
g) Any Elementary	--
h) Emergency	--
i) Other	--

10. Please check the manner in which you were appointed to your current mathematics teaching position (even for only one math section).

a) Transferred from non-math department within school or district. --
b) Transferred from another math department within district. --
c) New contract (with authorizing math credential). --
d) New contract (with emergency credential). --
e) Temporary or long-term substitute appointment. --
f) Other --

11. Please check the percent of your current instructional schedule spent teaching mathematics?

0--20 --
21--40 --
41--60 --
61--80 --
81-100 --

12. Please circle the total number of sections you currently teach daily for each of the two types of courses listed below (exclude computer classes).

a) 1st & 2nd year algebra, geometry and higher level courses. 1 2 3 4 5

b) Pre-alg/general/career/remedial/or other math courses. 1 2 3 4 5

Winter 1983-84

THE UC/CSU DIAGNOSTIC TESTS IN MATHEMATICS

Background

With science and technology continuing to play an increasingly important role in society, it has become necessary that all university students, not just those who show special talent or interest, understand mathematics and science. Consequently, an increasing number of academic departments require calculus and science courses as part of their majors. Unfortunately, many high school students, even those taking three or four years of mathematics courses, do not fully anticipate the level of preparation required for the beginning college mathematics and science courses, and some arrive at the university without the skills needed. Unaware of their mathematics skills deficiencies, they enroll in courses or programs in mathematics and science for which they are unprepared. Many do poorly and decide not to continue in their chosen course of study or major; some may drop out of school entirely.

Recognizing that the inadequate mathematics preparation of entering freshmen posed severe academic problems for students in both California university systems, CSU and UC formed a workgroup composed of faculty from the two systems to work to meet the following objectives:

1. To determine the mathematics competency areas necessary for success in calculus.
2. To develop diagnostic tests for these competency areas.

3. To determine the appropriate mastery levels in these areas for success in the university courses.
4. To suggest and design curricula to remove deficiencies identified by the examinations.
5. To develop a communication network with the high schools in California to provide topic-by-topic information as to the performance of their students on all of the competency-based tests devised by the workgroup and, in consultation with the high schools, to recommend resulting curricula changes.

The initial group consisted of four mathematicians and four physical scientists. Subsequently, representatives from the high schools and from the community colleges were added, along with a statistical analyst and an administrative coordinator.

The Tests and Their Purposes

Diagnostic mathematics examinations were designed to assess students' preparation for introductory college courses in Calculus and Precalculus.

The test designed to assess a student's readiness to enter Calculus is called the Precalculus Exam (available in 60-item, 90-minute and 40-item, 60-minute versions) and diagnoses competency in the following eight topics usually mastered by the end of the high school course in trigonometry:

1. Elementary operations with numerical and algebraic fractions
2. Operations with exponents and radicals
3. Linear equations and inequalities
4. Polynomials and polynomial equations
5. Functions
6. Trigonometry
7. Logarithmetic and exponential functions
8. Word problems

Students who do well on the Precalculus Exam (70% correct in each topic area) have demonstrated in field-testing studies their readiness for both the physical sciences and the life/social sciences calculus courses. Students who get 60% correct are advised to enroll in the calculus course of their choice but are cautioned that they may need to review some of the topics covered in the exam. If students score at the 50% level, they are recommended to take precalculus before the physical science calculus course and that extensive review may be necessary for success in the life/social sciences calculus course. If students score below 50% on the exams, they are strongly urged to take a precalculus course before they go on to either calculus course.

Once a student has decided to take a precalculus course, either by self-selection or as a result of a low score on the Precalculus Exam, it still remains to determine that student's level of competency in algebra. The Intermediate Algebra Exam (45-item, 45-minute) is designed to advise students as to where in the precalculus sequence

they should start and to diagnose strengths and weaknesses in the following eight topics usually mastered by the end of the high school Algebra II course:

1. Elementary numeric & algebraic operations
2. Rational expressions
3. Exponents and radicals
4. Linear equations; inequalities; absolute value
5. Polynomials; quadratic equations
6. The coordinate plane & graphing
7. Functions and logarithms
8. Word problems

Students scoring below 40% on the Algebra Exam are strongly urged to take a course in college algebra before they go on to precalculus Experience at both CSU and UC campuses has shown that approximately one half students who score below 50% on the Intermediate Algebra Exam fail to complete the precalculus course successfully if they do not first take a course in college algebra.

In addition to the Precalculus and Intermediate Algebra Exams, the workgroup designed a 50-item, 45-minute Basic Algebra Exam which may be used at the end of the junior or senior high school Algebra I course. This test covers the following nine topics:

1. Arithmetic
2. Polynomials
3. Linear equations & inequalities
4. Quadratic equations
5. Graphing
6. Rational expressions
7. Exponents and square root

8. Geometric measurement
9. Word problems

How the Tests Were Developed

The workgroup, wishing to diagnose the competencies necessary for success in calculus, rejected the idea of adopting or developing a normed test that might somehow measure levels of mathematical ability. Such aptitude or ability-based tests can be used as effective screening devices to allow only the most able students into the mathematics and science courses. If the hurdle is set high enough, then those that get over it are almost bound to succeed. This kind of screening is not what is needed. The workgroup instead sought to identify those skills and competencies learned and developed in high school that were necessary to survive in a college calculus course. Consequently, two kinds of items were avoided--so called "trick" questions that required significant cleverness to solve and "wish list" questions that required skills not absolutely necessary for success. The purpose of these tests is not to pick the most talented students for admissions to mathematics classes, but to identify those deficiencies that would almost certainly lead a student to drop out or to fail.

The first version of the Precalculus Test was constructed in the summer of 1978 with the help of a consultant, who is an expert in test design. In the summer of 1979 the initial version of the Intermediate Algebra Exam was constructed, and the Precalculus Exam was revised. The Basic Algebra Exam was first constructed in 1980. After an initial field testing of an exam, items are reviewed and revised.

Predictive Validity

Although the main purpose of these exams is to diagnose deficiencies, they also serve the purpose of predicting future success or failure in mathematics and science courses. Thus, one very good measure of the exams' effectiveness is the correlation between test scores and subsequent course grades. Such correlations were calculated at many campuses and for several types of mathematics courses. In general, these exams were found to be very good predictors of student performance in mathematics courses. For example, when the predictive validity of the Precalculus Exam is compared with more traditional predictors of student performance such as high school GPA or SAT scores as in Table 1, we find that the tests are considerably stronger predictors.

Table 1

Predictive Validity of 60-Item Precalculus Test at UCLA (Simple r^2)

	Physical Sciences Calculus	Life/Social Sciences Calculus
Total Test Score	.368	.244
SAT-Verbal	.033	.004
SAT-Math	.095	.080
High School GPA	.055	.222

Correlation coefficients do not give a complete picture of what student performance is really like. Table 2, below, shows the actual distribution of Calculus course grades of students scoring at four different levels on the Precalculus Exam.

Table 2

PERFORMANCE IN CALCULUS (MATH 1A)

COMPARED TO SCORES ON 60-ITEM PRECALCULUS EXAM

U.C. BERKELEY FALL, 1982

Exam Scores	Group I (43-60)	Group II (34-42)	Group III (25-33)	Group IV (0-24)	Math 1A Overall
Number of Students	236	123	47	26	1189
Average Grade in Course	3.14	2.60	2.17	1.65	2.69
Grade received:					
A	36%	12%	2%	0%	22%
B	45%	44%	40%	12%	39%
C	17%	37%	38%	50%	29%
D	2%	5%	11%	31%	6%
F	0.4%	1.6%	9%	8%	5%

Almost all analysis done to date on the predictive validity of these tests has been conducted on a representative student population, aggregating men, women, and minorities. In order to assess whether these tests were impeding women and minorities in an unfair way, a group of such students was monitored at the UC Berkeley campus. The physical science calculus course grades were correlated with the total test scores of 42 women and minority students participating in a special program. These students all scored 30 or above on the 60-item Precalculus Test; other students in the program who scored below 30 took precalculus or life science calculus courses. Even with the lower achieving group removed from the population, we still found a relatively high correlation between test scores and subsequent course grades, a simple r^2 of .36. This result is almost exactly what was observed at UCLA for a very large representative group of Freshmen, which included students of all backgrounds. As seen in Table 1, the simple r^2

for the UCLA students was .37. The actual performance of the women and minority students at Berkeley is given below in Table 3.

Table 3

Performance of Women and Minority Students in Physical Science Calculus as Related to Precalculus Exam Score

UC Berkeley, Fall, 1980

Score on 60-item Precalculus Exam	No. of students	Performance in Physical Science Calculus				
		Drop	D,F	C	B	A
50-60	13	0	0	0	2	11
40-49	15	0	2	4	5	5
Below 40	14	1	6	4	2	1

How the Tests Are Administered

Students should be made aware that scores on these exams are not part of their student records or course grades. They should be encouraged to do their best, but not to make wild guesses to raise their scores. Students should realize that if their scores overestimate their own mathematics competencies, they may find themselves taking a course for which they are ill-prepared.

The most accurate results can be obtained when students review and prepare for these tests in advance. By review, we do not mean to cram for the test, but rather to sharpen skills and memories that may have dulled with time. Review is often not needed for high school students, but many college freshman have not used their mathematics skills for awhile. For these students especially, we recommend that they be informed in advance as to the topics covered

on the tests and that they be encouraged to do some reviewing. Typical questions from each of the competency areas for both the Intermediate Algebra and Precalculus Exams are included in the Test Packet.

How the Tests Are Scored

Several optical scanning scoring systems have been developed: Scantron-based systems, designed primarily for high school and community college users, and an NCS-based system, designed with university campuses in mind. All systems have the same output format (sample enclosed) showing individual student performance by topic and total score, and class performance by item, topic, and total score. By special arrangement with the Mathematics Diagnostic Testing Project, separate, one-page reports can be provided for each individual student in addition to the class reports normally produced.

In the case of the high school and community college users, answer sheets are to be mailed to UC Berkeley, UCLA, UC San Diego, or California State University, Sacramento for scoring. Results will be returned within ten days. More detailed information regarding scoring can be found in the Test Administration Instruction Manual.

Universities may either score the tests at their own campus or mail the answer sheets to the NCS scoring center at CSU Sacramento, depending on the hardware/software available at the campus. Scoring service capabilities similar to those at CSU Sacramento have been developed at several other CSU campuses, and arrangements might be made to have a nearby campus do the scoring.

Typical Questions from Each of the Competency Areas of the Basic Algebra Ex

1. Arithmetic.

   $(0.12)^2$ =

   (A) 0.00144 (B) 0.0144 (C) 0.144 (D) 0.24 (E) 1.44

2. Polynomials.

   One of the factors of $x^2 - x - 6$ is

   (A) $x + 3$ (B) $x + 2$ (C) $x - 1$ (D) $x - 2$ (E) $x - 6$

3. Linear equations and inequalities.

   If $6x - 3 = 8x - 9$, then x =

   (A) -6 (B) -3 (C) 3 (D) $-\frac{6}{7}$ (E) $\frac{6}{7}$

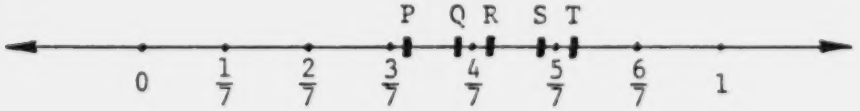
4. Quadratic equations.

   What are the possible values of x such that $3x^2 - 2x = 0$?

   (A) $-\frac{2}{3}$ only (B) 0 only (C) $\frac{2}{3}$ only (D) 0 and $\frac{2}{3}$ (E) $-\frac{2}{3}$ and $\frac{2}{3}$

5. Graphing.

   On the number line below, which letter best locates $\frac{5}{9}$?



   (A) P (B) Q (C) R (D) S (E) T

6. Rational expressions.

   $\frac{2}{w+1} - \frac{1}{w-1}$ =

   (A) $\frac{1}{w+2}$ (B) $\frac{1}{w^2-1}$ (C) $\frac{w-3}{w^2-1}$ (D) $\frac{w+3}{w^2-1}$ (E) $\frac{3w-1}{w^2-1}$

7. Exponents and square root.

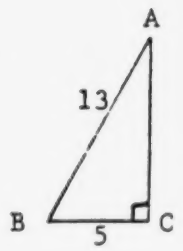
   If $x > 0$, then $\sqrt{64x^{16}}$ =

   (A) $8x^4$ (B) $8x^8$ (C) $16x^4$ (D) $32x^4$ (E) $32x^8$

page 2

8. Geometric measurement.

In the right triangle shown to the right, what is the length of AC?



(A) 8 (B) 12 (C) 18

(D) $\sqrt{18}$ (E) $\sqrt{194}$

9. Word problems.

If x is to 5 as y is to 8, what is the value of x when y = 2?

(A) $\frac{5}{16}$ (B) $\frac{4}{5}$ (C) $\frac{5}{4}$ (D) $\frac{16}{5}$ (E) 5